

# Progressive Expenditure Taxes and Growth

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## Abstract

Standard tax-policy advice suggests cutting income taxes, and relying more on expenditure taxes, to raise growth. The standard basis for this advice is endogenous growth theory, and the proof that an expenditure tax is preferred has involved a comparison of two proportional taxes in models with just one group of households. There is no formal evaluation of the concern that a proportional expenditure tax is regressive. Policy analysts have simply presumed that there would be support for a progressive expenditure tax, over a progressive income tax, if this option were formally modeled. This paper examines this presumption. Following Mankiw (2000), our simple endogenous growth model involves two groups. The first is an infinitely lived family dynasty that smooths consumption in the usual fashion. The second group is impatient and does not accumulate physical capital. Beyond the investment in human capital that is required to keep a job, these individuals simply consume their income (labour earnings plus a transfer payment). The transfer is financed by taxes levied on the other (richer) group. We find that both groups prefer that the transfer be financed by an income tax, not an expenditure tax. The intuition follows from the fact that redistribution creates a second-best initial condition, that is similar to that concerning government spending on infrastructure (discussed in Barro and Sala-i-Martin (1995)). They show that lower growth is preferred if government spending is fixed at a non-optimal proportion of the growing *GDP*, since growth accentuates this distortion. There is a similar distortion here. From the selfish point of view of the rich, any positive transfer is non-optimal, and growth magnifies this problem. Since the analysis does not support conventional wisdom, it challenges policy analysts to identify the analytical underpinnings of their views.

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## 1. Introduction

Many economists have called for a shift in tax policy: a decreased reliance on income taxation and an increased reliance on expenditure taxes (see, for example, Mintz (2001), Kesselman (2004) and the numerous references contained in these studies). The standard analytical underpinning for this tax-reform proposal is endogenous growth theory. While less emphasis is given to equity considerations (since standard growth theory involves a single representative agent), policy analysts sometimes call for a *progressive* expenditure tax – to avoid the regressivity that would otherwise accompany the use of sales taxes.

The purpose of this paper is to use a simple version of endogenous growth theory to review this debate. Two results emerge from this review. First, considering proportional taxes, if the optimal tax question is decided in a second-best setting in which government is “too big,” the analysis does not support elimination of the income tax. Many policy analysts argue *both* that government is “too big” and that income taxes are “bad.” The paper challenges these analysts to identify the analytical underpinnings of their views. The second result follows from an extension that involves two groups of households (as advocated by Mankiw (2000)). The first group pursues the standard consumption-smoothing strategy (as in Ramsey (1928)), so for this segment of the population, there is no difference from the earlier analysis. But the second group is more impatient. These individuals have a sufficiently high rate of time preference that they do not accumulate physical capital. They live “hand-to-mouth,” consuming all the income they receive in every period (beyond what is necessary to maintain the human capital that is required to be employed). This version of the model permits an analysis of a progressive expenditure tax; it does not provide clear support at all for the proposition that a progressive expenditure tax dominates a progressive income tax.

The remainder of the paper is organized as follows. In section 2, we explain the basic model and methodology, and we assess the relative appeal of proportional income taxes and expenditure taxes in a setting with no hand-to-mouth individuals. In section 3, we distinguish between physical and human capital, and we focus on hand-to-mouth households and progressive taxes. Concluding remarks are offered in section 4, and some technical derivations are reported in the Appendix.

## 2. Proportional Taxes

We begin with the proposition that total output produced each period,  $Y$ , is used in two ways. First, it is purchased by households to be consumed that period,  $C$ . Second, it is purchased by households to add to their stock of capital,  $K$ .  $\Delta K$  refers to that period’s increase in capital. The supply-equals-demand statement is

$$Y = C + \Delta K. \tag{1}$$

Next we specify the production process; in this initial case, it is very simple – output is proportional to the one input that is used in the production process,  $K$ :

$$Y = rK. \tag{2}$$

$r$  – the rate of return on capital – is the amount that output increases as additional units of the input are hired (the average and marginal product of capital).  $r$  is the pre-tax return that households earn on their saving. There are two potentially controversial aspects of this specification of the nation’s input-output relationship. First, there is no diminishing returns (as more capital is employed, its marginal product does not fall). The economy’s equilibrium growth rate is independent of fiscal policy if this assumption is not made, so we follow that standard practice here. Second, once the output emerges from this production process in the form of consumer goods, it is costless for society to convert that new output into additional capital. Again, this assumption is made to keep the paper consistent with standard analysis. Also, for simplicity, we ignore depreciation of capital.

In this initial specification, the government has just one function; it levies a proportional income tax rate,  $t$ , on the income that households earn by employing their capital, and a proportional sales tax rate,  $s$ , on household consumption spending. The government uses this revenue to finance transfer payments,  $R$ , to households. The government’s balanced-budget constraint is

$$R = trK + sC. \tag{3}$$

This specification of government is rather contrived, since all households are identical. There seems little point in having the government impose taxes that can distort each household’s decisions, when the only use for that revenue is to transfer the funds back to that same household. We make this assumption simply because it is standard in the literature. Also, it serves as a base upon which we can build a more interesting analysis of government policy in later sections of the paper.

The only other relationship that is needed to complete this initial analysis is the one that describes how households make their consumption-vs-saving decision. We assume that their objective is to maximize their utility, which is defined by the following function:

$$utility = \sum_{i=0}^{\infty} [1/(1+p)]^i [\ln C_i].$$

This utility function involves three features. First, at any point in time (indexed by  $i$ ), the *additions to utility* that accompany higher levels of consumption get smaller, the higher is the level of consumption initially. The logarithmic relationship is the simplest function that can impose this standard assumption of diminishing marginal utility of consumption. The second feature of the utility function is that it imposes the fact that households are impatient. Parameter  $p$  is the household’s rate of time preference; it is the discount rate that is used to capture the idea that the utility of future consumption is lower than that associated with current consumption. The third feature of the utility function is that it

involves households forming a plan that extends infinitely into the future. To rationalize this feature, it is useful to think of the household as an ongoing family dynasty.

Households are assumed to maximize utility subject to their budget constraint:

$$C + \Delta K = rK + R - trK - sC.$$

This constraint states that households pay for their consumption and capital accumulation (saving) by spending all their after-tax income and their transfer payment. We outline the formal solution of this constrained maximization in the Appendix. Here we focus on an intuitive interpretation of what constitutes optimal behaviour. To stretch their limited budget as far as possible, households must arrange their purchases so that the additional satisfaction (the marginal utility,  $MU$ ) that is had for each unit of income that needs to be paid, is identical at each point in time. For example,

$$(MU / price)_{present} = (MU / price)_{future}.$$

If this equality did not hold (say because the left-hand side exceeded the right-hand side), households could increase satisfaction by transferring spending from the future to the present. As they did so, the marginal utility of present consumption would fall and that for the future would rise. The payoff for further reallocation of spending would vanish when these changes in marginal utility just eliminated the original excess of the left-hand expression over that on the right.

To apply this general principle in the present setting, we pick the price of current consumption as the numeraire, and set this price equal to unity. Adopting this convention makes the price of future consumption equal to  $(1/(1+r(1-t)))$ , since – by waiting for a period to consume – households can earn the after-tax rate of interest on each unit of income. Given the logarithmic utility function, the marginal utility of present ( $i = 0$ ) consumption is  $(1/C_0)$  and the marginal utility of future ( $i = 1$ ) consumption is  $(1/((1+p)C_1))$ . Substituting these expressions into the optimum-consumption condition, we have:  $C_1 / C_0 = (1+r(1-t))/(1+p)$ . Subtracting unity from both sides, and simplifying by approximating  $(1+p)$  by unity, we end with

$$\Delta C / C = r(1-t) - p \tag{4}$$

The intuition behind this behavioural rule is straightforward. Households save as long as the after-tax return on capital exceeds their rate of impatience, and this saving generates the income necessary to have a positive percentage growth rate for consumption.

We now write the four numbered equations in a more compact form. We focus on a balanced-growth equilibrium in which consumption, output, and the capital stock all grow at the same rate:  $n = \Delta C / C = \Delta Y / Y = \Delta K / K$ , and with the tax rates,  $t$  and  $s$ , the ratio of consumption-to-capital,  $c = C/K$ , and the ratio of transfer payments-to-GDP,  $z =$

$R/Y$ , all constant. To accomplish this, we divide (1) and (3) through by  $K$  and  $Y$  respectively, and use (2) and the balanced growth assumption to simplify the results:

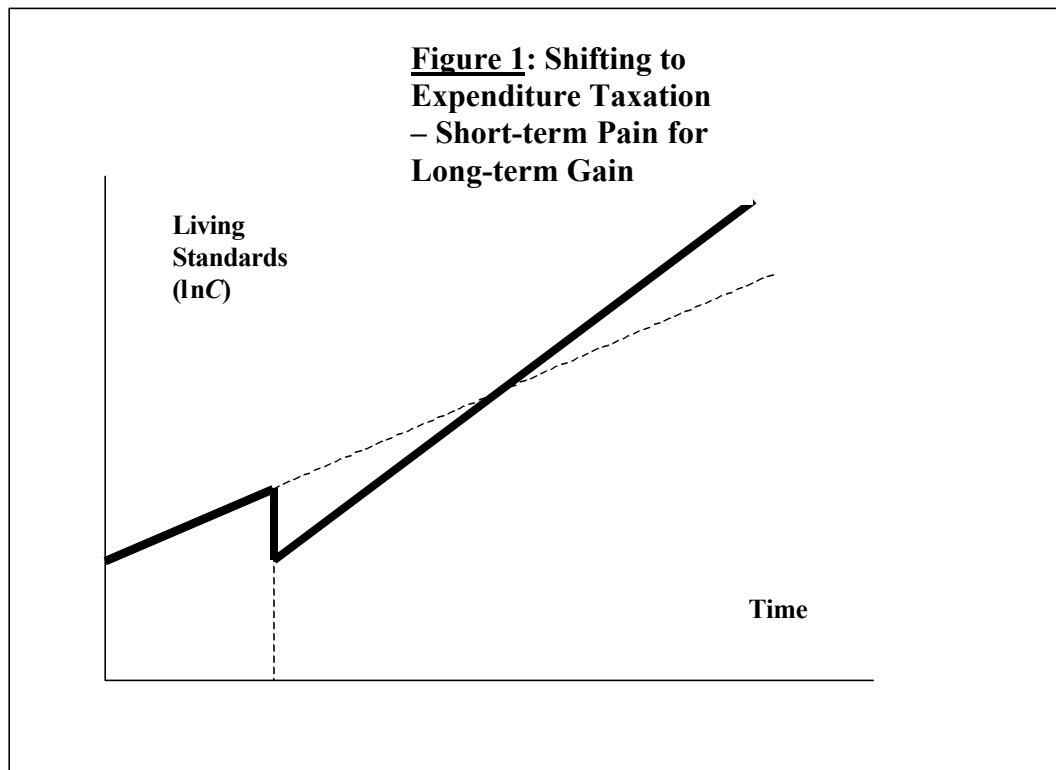
$$r = c + n \tag{5}$$

$$z = t + sc / r \tag{6}$$

$$n = r(1-t) - p \tag{7}$$

Equations (5) through (7) solve for the economy's growth rate,  $n$ , the consumption-to-capital ratio,  $c$ , and one of the tax rates (we assume that the sales tax rate,  $s$ , is residually determined by the model). The equations indicate how these three variables are affected by any change we wish to assume in the technology parameter,  $r$ , the taste parameter,  $p$ , and the government's exogenous instruments – the transfers-to- $GDP$  ratio,  $z$ , and the income tax rate,  $t$ .

Given the existing debate on tax reform, we focus on cutting the income tax rate, and financing this initiative with a corresponding increase in the sales tax rate. The effect on the growth rate follows immediately from (7):  $\Delta n / \Delta t = -r < 0$ . Thus, shifting to the sales tax raises the *ongoing growth rate* of living standards. But there is a *one-time level* effect on living standards (consumption) as well, and from (5) we see that this outcome is adverse:  $\Delta c / \Delta t = -\Delta n / \Delta t = r > 0$ . So a shift away from income taxation toward an increased reliance on expenditure taxes shifts the time path of household consumption in the manner shown in Figure 1. (Barro and Sala-i-Martin (1995), and others, have shown that this model involves no transitional dynamics; that is, the time path for living standards moves *immediately* from its original equilibrium path to its new equilibrium path.)



Given that there is short-term pain to achieve long-term gain, it is not immediately clear that this tax substitution is supported. But we can address this issue by using the household utility function. It is shown in the Appendix that when the present-value summation is worked out, overall social welfare ( $SW$ ) is

$$SW = [\ln C_0 + (n / p)] / p \quad (8)$$

where  $C_0 = cK_0$  and  $K_0$  is the initial capital stock, at the time when the tax substitution takes effect. From (8), we can determine the effect on overall welfare of the tax substitution:

$$\Delta SW / \Delta t = [(1 / c)(\Delta c / \Delta t) + (1 / p)(\Delta n / \Delta t)] / p$$

Using the one-time-level and ongoing-growth-rate effects reported above, we have

$$\Delta SW / \Delta t = r(p - c) / (cp^2)$$

which can be simplified by using (5) and (7):

$$\Delta SW / \Delta t = -(tr^2) / (cp^2) < 0.$$

The fact that this expression is negative indicates that the government can raise peoples' material welfare by cutting the income tax rate – all the way to zero. This is the standard proof that we should rely on expenditure, not income, taxes to finance the transfer payments. Because the generosity of the transfer does not affect the household's consumption-saving choice, there is no such thing as an “optimal” value for transfers. Whatever level is arbitrarily chosen has to be financed by expenditure taxes if the government wishes to maximize the material welfare of its citizens.

We now consider a sensitivity test, by asking how the optimal-tax conclusion is affected by our replacing the government transfer payment with a program whereby the government buys a fraction of the  $GDP$  and distributes it free to users (as in the case of government-provided health care). We continue to assume that no resources are needed to convert newly produced consumer goods into new capital or (now) into the government service. Thus, the economy's production function remains (2), and the supply-equals-demand relationship becomes

$$Y = C + \Delta K + G \quad (1a)$$

where  $G$  is the level of the government-provided good each period. The government budget constraint becomes

$$G = trK + sC. \quad (3a)$$

Finally, we assume that households value the government-provided good, so there are two terms in each period's utility function:

$$utility = \sum_{i=0}^{\infty} [1/(1+p)]^i [\ln C_i + f \ln G_i].$$

Parameter  $f$  indicates the relative value that households attach to the government service. Since the government imposes the level of government spending, individual households still have only one choice to make; they must choose their accumulation of capital with a view to maximizing the present discounted value of private consumption. The solution to that problem is still equation (4).

The model is now defined by equations (1a), (2), (3a) and (4). Defining  $g = G/Y$  as the ratio of the government's spending to  $GDP$ , these relationships can be re-expressed in compact form:

$$r(1-g) = c + n \tag{5a}$$

$$g = t + sc/r \tag{6a}$$

$$n = r(1-t) - p \tag{7}$$

and the modified overall material welfare function is

$$SW = [\ln C_0 + \ln G_0 + (1+f)(n/p)]/p \tag{8a}$$

It remains true that  $\Delta c / \Delta t = -\Delta n / \Delta t = r$ , but before we use these outcomes, we focus on the question of the optimal level of government program spending. We use the same principle that we applied to households above. In the present application (to the government's choice), that principle is to set government spending so that

$$(MU / price)_{private\ consumption\ good} = (MU / price)_{government\ provided\ good}.$$

Since the price of both goods is unity, this condition requires  $1/C = f/G$ ; that is, the optimal program-spending-to- $GDP$  ratio is  $g^* = fc/r$ . This definition is used to simplify the overall welfare effect of varying the income tax rate. The result is:

$$\Delta SW / \Delta t = [r^2 / (cp^2)][(g - g^*) - t].$$

As before, material welfare is maximized when this expression is zero, and this requires no income tax ( $t = 0$ ) only if government is set optimally ( $g = g^*$ ). If, however, government spending is "too big" ( $g > g^*$ ), there should be an income tax.

What is the intuition behind the result that income taxes should be used to finance some of the government when government is too big? The answer hinges on the fact that this

resource misallocation is in terms of a fixed *proportion* of a *growing GDP*. With growth, the magnitude of the distortion is being magnified. In this case, then, growth has a “bad” dimension. This is why it becomes “good” to rely, to some extent, on the anti-growth instrument for raising tax revenue (the income tax rate). Barro and Sala-i-Martin (1995, pp 144-161) consider a very similar model. But their government-provided good is “infrastructure,” and it enters the production function instead of the utility function. Nevertheless, the same dependence of the optimal tax question on second-best considerations – whether the government has set its expenditure “properly” – is stressed.

We conclude that we cannot know whether the actual economy involves the “preferred” ratio of income to sales tax rates or not, unless we know whether the government is “too big” or not, and by how much. Many pro-growth policy analysts defend two propositions: (i) that income taxes are too high relative to sales taxes, and (ii) that the government sector is too big. Our analysis has shown that the more correct these analysts are concerning the second proposition, the less standard growth theory supports their first proposition. One purpose of this paper is to identify this tension that policy analysts do not appear to be aware of.

### 3. Progressive Taxes

Thus far, there has been just one input in the production process, and we have interpreted that input quite broadly – as including both physical and human capital. For the remainder of the paper, we wish to contrast patient households who plan inter-temporally and acquire physical capital (as discussed above) with households that always spend their entire labour income and transfer payments. To make this comparison explicit, we must now distinguish physical capital ( $K$ ) from human capital ( $H$ ). The supply-equals-demand equation becomes

$$Y = C + \Delta K + \Delta H + G$$

since output is either consumed privately, consumed in the form of a government-provided good, or used to accumulate physical and human capital.

Both forms of capital, the government service, and consumer goods, are produced via the following production function:

$$Y = aK^b H^{1-b}$$

with  $b$  a positive fraction. By defining  $B = H/K$  and  $A = aB^{1-b}$ , the production function can be re-expressed as  $Y = AK$  – the same form as that used in Section 2 above. Barro and Sala-i-Martin (1995, pp 144-146) suggest this interpretation for the standard  $AK$  production function.

Compared to the tax-policy literature, this specification of the production function involves a “middle-of-the-road” assumption concerning the factor intensities that are

involved in the production process. The standard  $AK$  model of endogenous productivity growth involves a “short cut” by assuming that the accumulated stock of knowledge is simply proportional to the aggregate stock of *physical* capital. In contrast to this, the basic version of Lucas’ (1988) two-sector framework makes the accumulation of knowledge completely independent of the stock of physical capital. The “education” sector is assumed to involve only the pre-existing human capital in the production of new knowledge, so it is *human* capital, not physical capital, that is the engine of productivity growth. The standard  $AK$  specification leads to the policy proposition that interest-income taxes should not exist, while the assumption that only human capital is needed to acquire more knowledge leads to the proposition that wage-income taxes should not exist. The specification used here is an appealing intermediate specification, since it implies that neither form of income tax is preferred to the other. This is because it involves the assumption that physical capital and human capital are used in the same proportions when producing all items: consumption goods, new physical capital goods, and new knowledge (human capital). We regard this as an appealing baseline assumption for evaluating the desirability of income vs. expenditure taxes.

It is assumed that households rent out their physical and human capital to firms (that are owned by other households). Profit maximization on the part of firms results in factors being hired to the point that marginal products just equal rental prices ( $r$  is the rental price of physical capital;  $w$  that for human capital):

$$bA = r$$

$$(1-b)Y/H = w.$$

Households maximize the same utility function, this time subject to:  $(1+s)C + \Delta K + \Delta H = r(1-t)K + w(1-t)H$ . In addition to the now-familiar consumption-growth relationship,  $n = r(1-t) - p$ , the optimization leads to  $r = w$ . In equilibrium, households must be indifferent between holding their wealth in each of the two forms of capital, and the equal-yields relationship imposes this equilibrium condition. Finally, the government budget constraint is  $G = trK + twH + sC$ , and the compact version of the full model is:

$$\begin{aligned} A &= aB^{1-b} \\ r &= bA \\ n &= r(1-t) - p \\ B &= (1-b)/b \\ n(1+B) &= A(1-g) - c \\ g &= t + sc/A. \end{aligned}$$

The endogenous variables are:  $A$ ,  $B$ ,  $r$ ,  $n$ ,  $c$ , and one of the tax rates. It is left for the reader to verify that all the conclusions of Section 2 continue to hold in this slightly extended setting. As suggested above, the reason that the distinction between physical and human capital does not yield different conclusions is because the economy uses the two forms of capital in exactly the same proportions when producing all four items:

consumer goods, the government good, new physical capital goods, and new knowledge (human capital).

Mankiw (2000) has suggested that, for the sake of realism, all fiscal policy analyses should allow for roughly half of the population operating as infinitely-lived family dynasties and the other half operating hand-to-mouth. In the previous section of this paper, we did not follow this advice. Further, since all households have been identical, progressive taxes could not be considered. We follow Mankiw’s suggestion in this section of the paper, both to increase the empirical applicability of the analysis, and to make it possible to investigate progressive taxes.

Since the poor consume a higher proportion of their income than do the rich, a shift from a proportional income tax to a proportional expenditure tax makes the tax system regressive. For this reason, policy analysts are drawn to the progressive expenditure tax. It is hoped that this tax can avoid creating an equity problem, as we take steps to eliminate what is perceived to be an efficiency problem. In this section of the paper, we use our otherwise standard growth model to examine shifting between income and expenditure taxes – when both taxes are strongly progressive. To impose progressivity in a stark fashion we assume that only “Group 1” households – the inter-temporal consumption smoothers who are “rich” – pay any taxes. “Group 2” households – the “poor” individuals who live hand-to-mouth – pay no taxes. In this section, we revert to our original specification concerning the expenditure side of the government budget. We assume that there is no government-provided good; the taxes collected from the rich are used to make a transfer payment to the poor.

Group 2 households have a utility function just like the Group 1 utility function, except Group 2 people are more impatient. Their rate of time preference is  $q$ , which is sufficiently larger than the other group’s rate of impatience,  $p$ , that Group 2 people never acquire any physical capital. It is assumed that they simply have to do some saving, in the form of acquiring the human capital that is absolutely required for employment. But beyond that “compulsory” saving, they do none. Thus, this group’s consumption function is simply their budget constraint. Using  $E$  to denote total expenditure by this portion of the population (assumed to be one half of the total), we have  $E = R + (wH - \Delta H)/2$ , since this group receives no interest income and it pays no taxes, but it does receive the transfer payment. Since only Group 1 pays either tax, both the income and the expenditure tax are progressive. The government budget constraint is  $R = t[rK + (wH / 2)] + sC$ . We re-express the Group 2 expenditure relationship and the government budget constraint by using the optimization conditions for firms and the rich households, and by defining  $e = E/K$ . The compact form of the model is:

$$\begin{aligned}
 A &= aB^{1-b} \\
 r &= bA \\
 B &= (1-b)/b \\
 n &= r(1-t) - p \\
 n &= r - b(c+e) \\
 e &= zA + [(1-b)A - Bn]/2
 \end{aligned}$$

$$z = ((1+b)t/2) + (sc/A)$$

We assume that the initial situation involves only an expenditure tax ( $t = 0$  initially), and we use the model to determine the effects of moving away from this initial situation by introducing the income tax (and cutting the expenditure tax by whatever is needed to maintain budget balance). If the conventional wisdom (that a progressive expenditure tax is preferred to a progressive income tax) is to be supported, the analysis must render the verdict that undesirable effects follow from the introduction of the income tax. The results are

$$(\Delta n / \Delta t) = -r < 0$$

$$(\Delta c / \Delta t) = r(1+b)/2b > 0$$

$$(\Delta e / \Delta t) = r(1-b)/2b > 0$$

As usual, moving toward a heavier reliance on the income tax brings favourable consumption-*level* effects (in this case for both groups), but an unfavourable growth-*rate* effects. As above, we combine these competing effects in overall welfare calculations. After some simplification, using the initial conditions, for Group 1, we have

$$(\Delta SW_1 / \Delta t) = [r/2p^2bc][p(1+b) - 2bc]$$

and for Group 2, we have

$$(\Delta SW_2 / \Delta t) = [r/2q^2be][q(1-b) - 2be]$$

The model must be calibrated to assess the sign of these overall material welfare effects. To illustrate the outcomes, we assume the following illustrative values:

Physical capital's share of <i>GDP</i> ( $b$ )	33%
Rate of return on capital	12%
Growth rate	2%
Initial income tax rate	0%
Initial sales tax rate	10%
Total consumption-to- <i>GDP</i> ratio	80%
Group 1 consumption-to- <i>GDP</i> ratio	48%
Group 2 consumption-to- <i>GDP</i> ratio	32%
Investment-to- <i>GDP</i> ratio	20%

These values imply a rate of impatience for the “poor” that is twice that of the “rich” ( $p = 10\%$ ), and the following initial conditions:  $c = 0.182$  and  $e = 0.118$ . When these representative parameter values are substituted into the welfare change expressions, we see that both are positive. This implies that both rich and poor are made better off by having the lower growth rate that results from replacing the progressive expenditure tax

with the progressive income tax. While other calibrations might lead to the opposite conclusion, and this additional quantitative analysis needs to be done and reported in the next draft of the paper, we can surely conclude that, at the very least, there is limited support for conventional wisdom.

The intuition behind this finding runs as follows. From the selfish perspective of the rich, the optimal transfer to the poor is zero. Thus, with a positive transfer, the government is “too big,” and since this problem is a constant proportion of a growing *GDP*, less growth is preferred. This is another application of the general principle that emerged in Section 2 of the paper. Finally, as far as the poor are concerned, in this illustrative calibration, they are impatient enough to prefer the favourable consumption-level effect that accompanies the shift to the income tax.

#### **4. Conclusions**

The simplest version of standard growth theory does not provide a solid basis for the common propositions that have been advanced by many policy analysts. The standard advice is that the income tax should be replaced by an expenditure tax (whether or not these taxes are progressive). Since our analysis does not support these propositions, and since it is an entirely standard version of endogenous growth theory, the results represent a challenge for policy analysts – to clarify the analytical underpinnings of their advice.

## Appendix

For the section 2 analysis, the household optimization involves maximizing

$$\int_0^{\infty} [\ln C + f \ln G] e^{-pt} dt$$

subject to

$$C + \dot{K} = r(1-t)K - sC$$

by choosing  $C$  and  $K$ . The first-order conditions can be derived by standard methods. They have been written in a more user-friendly discrete-time format in the text. Note that the formal derivation does not require the approximation that was part of the intuitive discussion in the text.

Again, for the section 2 analysis, the social welfare function is the utility function of the representative household:

$$SW = \int_0^{\infty} [\ln C + f \ln G] e^{-pt} dt.$$

We know that  $C_t = C_0 e^{nt}$  and that  $G_t = G_0 e^{nt}$  so the social welfare expression can be simplified to

$$SW = [(\ln C_0 + f \ln G_0) / p] + (1 + f) \int_0^{\infty} \ln(e^{nt}) e^{-pt} dt.$$

The integral in this last equation can be re-expressed as  $n \int t e^{-pt} dt$ , and this, in turn, can be simplified by using the standard formula  $\int u dv = uv - \int v du$ , to yield

$$(-1/p)[t + \int -e^{-pt} dt] = -e^{-pt} [(1 + pt) / p^2]_0^{\infty}.$$

When this solution is evaluated at the extreme points in time, and the result is substituted into the expression for  $SW$ , equation (8a) in the text is the result. When differentiating that equation with respect to the tax rate, we use  $c = C/K$ , which implies that  $(dC_0 / C_0) dt = (dc / c) / dt$  since  $K$  cannot jump ( $K_0$  is independent of policy). Also, since  $G_0 = grK_0$ , we know that  $dG_0 / dt = 0$  as well.

The  $g^* = fc / r$  result emerges from maximizing  $SW$  subject to the resource constraint, that is, by evaluating  $dSW / dG = 0$ .

The derivations for section 3 of the paper follow in a similar fashion.

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