Inflation and Real Wage Dispersion

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Abstract

Linkages between inflation and within-skill real wage dispersion are studied in a cash-in-advance economy. A general equilibrium monetary model featuring Walrasian goods market but a non-Walrasian labor market is constructed. Labor market is characterized by search frictions. In the labor market, firms post wages and both employed and unemployed workers search among the posted wages. Workers and firms are matched through a matching function. In such an environment, the paper shows that a unique stationary monetary equilibrium with non-degenerate real wage distribution exists. Moreover, consumption, output, employment, and real wages are lower in such an equilibrium compared to the competitive case when there is no friction in the labor market. In addition, an increase in the inflation rate increases the real reservation wage and the average real wage offer and lowers the variability in real wages. An increase in the inflation rate also reduces consumption, output, vacancy, and employment levels. The Friedman rule is not optimal. Because of monopsony power of firms in the labor market, the optimal inflation rate exceeds the discount rate.

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1. Introduction

What is the effect of inflation on real wage dispersion? This paper studies linkages between inflation and within-skill real wage dispersion in a cash-in-advance economy.

Wage dispersion is one of the most important determinants of income dispersion. In addition, wage dispersion affects the allocation of resources in the economy through its effect on the search behavior of workers and the level and the composition of the vacancies created by firms. Empirical literature shows that within-skill wage dispersion accounts for sixty to seventy percent of the total wage dispersion in the industrialized countries (OECD 1997, Devroye and Freeman 2001). Given the fact that within-skill wage dispersion explains a large part of overall wage dispersion, the question of linkages between inflation and within-skill real wage dispersion is of considerable policy significance.

Inflation by affecting within-skill wage dispersion can affect both the allocation of resources as well as income distribution in an economy. To the best of the author's knowledge there is no general equilibrium monetary model which studies linkages between inflation and within-skill wage dispersion.[1] The present paper is an attempt to fill this gap.

In the paper, a general equilibrium cash-in-advance economy featuring a Walrasian goods market but a non-Walrasian labor market is constructed. In the goods market, buyers meet sellers instantaneously and buy goods subject to a cash-in-advance constraint. Labor market, in contrast, is characterized by friction. Workers and producers have to search for suitable matches and they are matched through a matching function. Labor market is modeled using a version of wage posting model with on-the-job search developed by Mortensen (2000). In the labor market, producers have monopsony power and make take-it-or-leave it offer to workers when they meet.

In such an economy, the paper shows that a unique stationary monetary equilibrium with non-degenerate real wage distribution exists. Moreover, consumption, output, employment, and real wages are lower in such an equilibrium compared to the competitive case where there is no friction in the labor market.

The paper finds that in equilibrium an increase in the inflation rate leads to lower within-skill real wages dispersion. The paper also finds that in the stationary monetary equilibrium with dispersed real wages, an increase in the inflation rate reduces consumption, output, vacancy, and employment levels through standard inflation tax. In addition, the paper finds that the Friedman rule is not optimal. Because of monopsony power of firms in the labor market, the optimal inflation rate exceeds the discount rate.

In the present paper, I embed a wage posting model with on -the-job search developed by Mortensen (2000) in a cash-in-advance economy. Mortensen (2000) shows that in a

[1] Head and Kumar (2002) study the effect of inflation on real price dispersion. The model can be recast in terms of the effect of inflation on within-skill real wage dispersion. However, the model is full employment model and the matching probabilities are not affected by inflation. We will see that the effect of inflation on the matching probabilities has important implications for both within-skill real wage dispersion and other macro-economic variables.

model with homogeneous firms and workers wage dispersion can be an equilibrium outcome in the labor market with friction where both employed and unemployed workers search. In the model, unemployed workers search for jobs which pay them at least their reservation wage. Employed workers search for jobs which pay higher wages than what they currently earn. In the model, firms are induced to post wages other than the reservation wage of unemployed workers because posting of higher wages helps to attract workers from low wage jobs and simultaneously reduces workers' turnover. Embedding Mortensen model in a cash-in-advance economy allows me to study the linkages between inflation and within-skill real wage dispersion.

In the model, inflation through standard inflation tax increases the real reservation wage; the real wage at which unemployed workers are indifferent between working and not working. An increase in the real reservation wage reduces the monopsony power of firms leading to reduction in real wage dispersion. An increase in the real reservation wage also reduces the equilibrium level of vacancy resulting in a fall in employment, output, and consumption.

The economy has two sources of inefficiency- i) the cash-in-advance constraint and ii) the monopsony power of firms. The Friedman rule removes the first source of inefficiency but not the second. Hence, the optimal rate of inflation in this economy exceeds the Friedman rule.

The rest of the paper is organized as follows. Section 2 describes the model. In section 3, the optimal strategies of the households are characterized. Section 4 defines a class of stationary monetary equilibrium and derives conditions for the existence of a unique stationary monetary equilibrium with non-degenerate real wage distribution. Section 5 studies the optimal inflation rate. In section 6, the effect of inflation rate on equilibrium variables are studied. An example is also constructed to illustrate the mechanisms through which inflation rate affects equilibrium values. This is followed by concluding remarks. All proofs of propositions are contained in the appendix.

2. The Economy

2.1 The Household Structure

Time is discrete and continues forever. Consider a single commodity cash-in-advance economy without any aggregate uncertainty comprising of a large number of identical households with the measure normalized to unity. Each household, in turn, is large and comprised of three types of infinitely-lived members- a buyer, a firm, and a large number of identical workers.[2] Workers are either employed or unemployed. Let the measure of workers in a household be unity.[3]

[2] Firms need not be part of the household. One can assume that firms are owned by the households and firms take decision in order to maximize the utility of the owners.

[3] The construction of the household is similar to that in Shi (1999) and Head and Shi (2000). This construction makes the model highly tractable analytically (see below).

Each type of members in a household plays a distinct role. The buyer buys desired consumption good in the goods market. The firm hires workers in the labor market, produces, and sells the good in the goods market. Unemployed workers search for suitable jobs. Employed workers work and also search for better jobs (i.e. on-the-job search is allowed in the model). The members of the household do not have independent preferences. Rather, the household prescribes the trading strategies for each member which maximize the household utility. The members of the household share equally in the utility generated by the household consumption.

2.2 Trading

Trades in this economy take place in two separate markets- the goods market and the labor market. It is assumed that during any period t, trading takes place first in the goods market and then in the labor market. [4] In the goods market, the buyers use cash to buy goods subject to a cash-in-advance constraint and sellers sell their produced good for cash. The goods market, as in the standard cash-in-advance economy, is assumed to be competitive.

The labor market, however, is characterized by friction. The match between workers and producers are not instantaneous. Rather, the producers who want to hire workers and workers who either want to work or change their jobs have to search for suitable matches. The search is assumed to be sequential. Workers and producers are matched randomly through a matching function.

It is assumed that the producers face monopsonistic competition in the labor market and they post wage trajectories (i.e. they make take it or leave it offer to the workers). Workers (both employed and unemployed) search among the posted wage trajectories. Because of random matching, producers and workers face idiosyncratic risks in the matching outcomes. In particular, random matching induces a non-degenerate distribution of money holdings. In addition, the model generates non-degenerate wage earnings distribution which also induces a non-degenerate distribution of money holdings. The construct of large household makes the distribution of money holding degenerate and allows the use of representative household which makes the model highly tractable.

In the model, employed workers of the household and employees of the firm can be heterogeneous with respect to the wage trajectories. Table 1 lists the notation of the workers and the employees of the firm in the representative household.

[4] The results of the model do not hinge on whether goods market opens first or the labor market. What is important is that there be time separation between when the households receive money and when they spend it. For the inflation tax (which is the focus here) to have impact, the time-separation between the acquisition of money and spending is essential.

Table 1
Employed Workers and Employees in a Household

	Employed Workers	Employees of the Firm	Unemployed Workers
Measure at Wage Trajectory W_t	$e_t(W_t)$	$J_t(W_t)$	
Set of Wage Trajectories	$S_{EW}(t)$	$S_{JW}(t)$	
Total Measure	$e_t = \int_{S_{EW}(t)} e_t(i)di$	$J_t = \int_{S_{JW}(t)} J_t(i)di$	u_t

In the rest of the section, I describe in detail the functioning of the goods market and the labor market and the decision variables of the representative household. As a convention, the decision variables of other households which are taken as given by the representative household are denoted with superscript '^'.

2.2.1 Goods Market

Let M_t be the available nominal money balance with the representative household at the beginning of period t. At the beginning of period t, the household allocates the available money M_t to the buyer who goes to the goods market to acquire the consumption good subject to the cash-in-advance constraint

$$p_t c_t \leq M_t \quad \forall\ t \tag{2.1}$$

where p_t is the price of the consumption good and c_t is the amount purchased. The firm in the household produces the consumption good using the employees and sells the good in the goods market. For simplicity, it is assumed that the firm uses linear production technology and each employee produces y units of good. Since, the measure of employees in the firm is J_t at time t, the total receipt of nominal money for the firm at time t is yp_tJ_t.

2.2.2 Labor Market

At the beginning of period t, the representative household decides the measure of vacancies to be created, the associated set of wage trajectories (which can be singleton), and the optimal job-acceptance strategies of both employed and unemployed workers. After the decisions of the household, firm and workers go to the labor market.

The firm of the household posts vacancies v_t with the associated set of wage trajectories $S_{PW}(t)$. A wage trajectory W_t is a vector of nominal wages which the firm agrees to pay to the matched worker, $W_t \equiv \{w_{t+1},\ w_{t+2},\ w_{t+3}....\}$. In the case of a match, it pays the employed worker w_{t+1} at time $t+1$, w_{t+2} at time $t+2$, and so on as long as the match continues. A match can get destroyed either due to idiosyncratic shocks or the employed worker leaving for a better job. The idiosyncratic shocks arrive exogenously at the Poisson rate of ρ per unit of time. It is assumed that idiosyncratic shocks destroy only the existing jobs matches and not the newly formed ones.

For posting a vacancy, the household incurs disutility k per unit of time. It is assumed that for each job, the firm hires only one worker. Also, as is common in search literature, it is assumed that the firm treats each vacancy separately from other vacancies. It is also assumed that the firm pays its workers wages from the current receipt i.e. the wage payment is not subject to a cash-in-advance constraint.[5] Once a vacancy gets matched, the match starts producing from the next period which continues till the match gets destroyed.

In the model, both employed and unemployed workers search among the posted wage trajectories. For simplicity, let the search-intensity of both employed and unemployed workers be equal and normalized to one.[6] Also suppose that the disutility cost of search is unity.

Before employed and unemployed workers go to the labor market and search, the household prescribes the job-acceptance strategies i.e. which wage trajectories to accept and which to reject. After receiving instructions regarding the job-acceptance strategies the workers go the labor market. It is assumed that while searching the workers know the distribution of wage trajectories posted but not the actual wage trajectory they will realize. The actual wage offer is revealed only after the search. After getting matched in period t, the newly matched workers start production in the next period. It is assumed that employed workers incur disutility ϕ per period from working. Because of idiosyncratic shocks employed workers also face the risk of unemployment. Employed workers can become unemployed with rate ρ per period.

2.3 The Matching Function

Workers and vacancies are matched through an aggregate matching function which relates the flow of hiring, s, to the aggregate measures of employed and unemployed workers searching and the aggregate measure of vacancies. It is assumed that the matching function $s(\hat{v}_t, \hat{e}_t, \hat{u}_t)$ is concave, increasing, and is subject to constant returns to scale. It is also assumed that searching employed and unemployed workers are perfect substitutes i.e only the aggregate measure of the workers searching matters and not their composition. The aggregate flow of matches then is given by

$$s(\hat{v}_t, \hat{e}_t, \hat{u}_t) = s(\hat{v}_t, 1) = s(\hat{v}_t) \tag{2.2}$$

Further assume that $\lim_{\hat{v}_t \to 0} s(\hat{v}_t) \to 0$ and $\lim_{\hat{v}_t \to 0} s'(\hat{v}_t) \to \infty$. Since, the aggregate measure of workers in the economy is unity, (2.2) also defines the aggregate matching rate of workers. The aggregate matching rate of a vacancy is given by

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[5] Since, the goods market is Walrasian, this assumption is not restrictive.

[6] One can endogenize the search-intensities of employed and unemployed workers. Endogenization of search-intensity will lead to considerable analytical and computational complexity. The reason is that the model generates a non-degenerate wage earnings distribution which will induce non-degenerate distribution of search intensities which is a high dimensional object.

$$\frac{s(\hat{v}_t)}{\hat{v}_t} \tag{2.3}$$

2.4 Preferences

The representative household maximizes the discounted sum of utilities from the sequence of consumption less the disutility cost arising from workers' working and searching and posting of vacancies by the firm.

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \left[u(c_t) - (1+\phi)e_t - u_t - kv_t \right] \tag{2.4}$$

where r is the rate of discount. Let $u'(c_t) > 0$ and $u''(c_t) \leq 0$.

3. The Household's Choice Problem

3.1 Timing

The representative household at the beginning of period t enters with e_t employed workers and their distribution, u_t unemployed workers, and J_t employees in the firm and their distribution. At the beginning of each period, the household receives lump-sum money transfer $(g-1)\hat{M}_t$ where $\hat{M}_t$ is the per household money stock at time t. The lump-sum transfer of money is added to the money balance carried from the previous period. The household gives the available money balance, M_t, to the buyer who goes to the goods market to purchase the consumption good. The firm of the household produces the good using the existing employees and goes to the goods market to sell the good. After trading in the goods market, the buyer comes back to the household with the purchased good and any residual money balance and the firm with the nominal sales receipt. The firm pays wages to its employees from the sales receipt. The employed workers return to the household with the nominal wage receipt. The profit of the firm, the wage receipt of the employed workers, and any residual balance brought back by the buyer are added to the household money balance for the next period.

Labor market opens up. The household decides the measure of vacancies to be posted v_t, the associated set of wage trajectories to be offered, $S_{PW}(t)$, and prescribes the job-acceptance strategies to the workers. Employed and unemployed workers search among the posted wage trajectories. Idiosyncratic shocks are realized. Trading in the labor market and idiosyncratic shocks decide the next period measure of employed workers e_{t+1} and their distribution, measure of unemployed workers u_{t+1}, and the measure of employees of the firm J_{t+1} and their distribution. At the end of labor market session, workers and firms go back to their respective households and consumption takes place. Time moves to the next period $t+1$.

3.2 The Optimal Job Acceptance Strategies of Workers

Before we formally set up the household optimization problem, it is convenient to discuss the optimal job-acceptance strategies of workers prescribed by the household. The objective of the household is to maximize the household utility. Since, the household has a large number of workers, the contribution of any worker to the household utility is negligible. Because of this, the household bases its job-acceptance strategies on the marginal value of employment.

Let $\Omega_{et}(W_t)$ be the marginal value of an employed worker to the household at wage trajectory W_t. Note that different wage trajectories can provide identical marginal value. Similarly, let Ω_{ut} be the marginal value of an unemployed worker to the household. It is optimal for the household to instruct an employed worker to accept any wage offer which provides higher marginal value to the household compared to the marginal value of the wage trajectory at which he currently works. In other words, an employed worker with wage trajectory W_t accepts any wage trajectory W_t' such that $\Omega_{et}(W_t)' > \Omega_{et}(W_t)$. Similarly, the household instructs unemployed workers to accept any wage trajectory W_t such that $\Omega_{et}(W_t) \geq \Omega_{ut}$.

Using the marginal values of employed workers, one can define probability distributions over the set of wage trajectories posted $S_{PW}(t)$ and the set of employed workers $S_{EW}(t)$. Let $F(\Omega_{et}(W_t))$ be the probability distribution of the marginal values of the wage trajectories posted by a household firm. Similarly, let $G(\Omega_{et}(W_t))$ be probability distribution of the marginal values of the wage trajectories earned by employed workers. In what follows, I will refer to $F(\Omega_{et}(W_t))$ as the probability distribution of marginal values posted/offered and $G(\Omega_{et}(W_t))$ as the probability distribution of marginal values earned.

3.3 The Household Optimization Problem

Taking the aggregate distribution of marginal values posted $\hat{F}(\Omega_{et}(W_t))$, the aggregate distribution of marginal values earned $\hat{G}(\Omega_{et}(W_t))$, the aggregate level of vacancy $\hat{v}_t$, the price level in the goods market p_t, the optimal choices of other households, and the initial conditions $\{\ M_0,\ e_0,\ u_0,\ \mathrm{G}(\Omega_{e0}(W_0))\}$ as given, the household chooses the sequence $\{c_t,\ M_{t+1},\ v_t,\ S_{PW}(t)\}\ \forall\ t \geq 0$ to solve the following problem

Household Problem (PH)

$$\max_{c_t, M_{t+1}, v_t, S_{PW}(t)} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \Big[u(c_t) - (1+\phi)e_t - u_t - kv_t\Big]$$

subject to the cash-in-advance constraint given in (2.1) and the laws of motion

$$M_{t+1} \leq M_t + (g-1)\hat{M}_t - p_t c_t + [p_t y J_t - \int_{i \in S_{JW}(t)} i_t J(i) di] + \int_{j \in S_{EW}(t)} j_t e(j) dj \quad (3.1)$$

where i_t is time t element of a wage trajectory. j_t has the same interpration.

$$J_{t+1}(W_{t+1}) \leq \Big[1 - \rho - s(\hat{v}_t)(1 - \hat{F}_t(\Omega_{et}(W_t)))\Big] J_t(W_t)$$

$$+\frac{s(\hat{v}_t)}{\hat{v}_t}\Big[\hat{u}_t + (1-\hat{u}_t)\hat{G}_t(\Omega_{et}(W_t))\Big]v_t(W_t) \ \ \forall W_t \tag{3.2}$$

$$e_{t+1}G_{t+1}(\Omega_{et+1}(W_{t+1})) \leq \Big[1-\rho - s(\hat{v}_t)(1-\hat{F}_t(\Omega_{et}(W_t)))\Big]e_t G_t(\Omega_{et}(W_t))$$

$$+s(\hat{v}_t)u_t\hat{F}_t(\Omega_{et}(W_t)) \ \ \forall W_t \tag{3.3}$$

$$u_{t+1} \leq u_t + \rho e_t - s(\hat{v}_t)(1-\hat{F}_t(\Omega_{ut}))u_t \tag{3.4}$$

(3.1) describes the law of motion of the household's nominal money balance. The first term in the right hand side is the nominal money balance of the household at time t, the second term is the lump-sum monetary transfer at the beginning of period $t+1$, and the third term is the money spent by the buyer at time t. The fourth term in the bracket is the nominal profit made by the firm at time t. The first integral is the total nominal wage payment made by the firm. The second integral is the total nominal wage payment received by the employed workers of the household.

(3.2) states the law of motion of the employees of the firm at different wage trajectories. The first term in the right hand side is the measure of employees of the firm at the wage trajectory W_t at the beginning of period t remaining with the firm at the end of period t. Workers can leave firm either due to idiosyncratic shocks or due to workers receiving better job offers. The second term in the right hand side is the measure of new matches at the posted wage trajectory W_t. The second term reflects the fact that a vacancy with wage trajectory W_t attracts all the unemployed workers and the employed workers with marginal values earned less than or equal to $\Omega_{et}(W_t)$. Notice that by posting wage trajectories which give workers higher marginal values the firm can reduce workers' turnover as well as increase the matching rate of vacancy by attracting larger pool of searching workers.

(3.3) specifies the law of motion of the distribution of marginal values earned by the employed workers of the household. The term in the left hand side is the measure of employed workers with marginal values earned less than or equal to $\Omega_{et+1}(W_{t+1})$ at the beginning of period $t+1$. The first term in the right hand side is the pool of employed workers with marginal values earned equal to or less than $\Omega_{et}(W_t)$ at the beginning of period t remaining in the pool at the end of period t. An employed worker leaves this pool either due to idiosyncratic shocks or the worker receiving a wage offer with marginal values higher than $\Omega_{et}(W_t)$. The second term is the measure of unemployed workers who receive wage offers of marginal values of $\Omega_{et}(W_t)$ and less.

(3.4) describes the law of motion of unemployed workers. The first term in the right hand side is the measure of unemployed workers in the household at the beginning of time t. The second term is the measure of employed workers becoming unemployed due to idiosyncratic shocks at time t. The third term is the measure of unemployed workers at time t becoming employed.

Let ω_{ct}, ω_{Mt}, and $\Omega_{Jt}(W_t)$ be the Langrangian multipliers associated with constraints (2.2), (3.1), and (3.2) respectively.

3.4 The Optimal Choice of c_t and M_{t+1}

The first order conditions for the optimal choices of c_t and M_{t+1} are given by

$$c_t: \ \frac{u'(c_t)}{p_t} = \omega_{ct} + \omega_{Mt} \tag{3.5}$$

$$M_{t+1}: \ \omega_{Mt} = \frac{1}{1+r}[\omega_{Mt+1} + \omega_{ct+1}] \tag{3.6}$$

The slackness condition associated with the optimal choice of consumption is given by

$$\omega_{ct}[M_t - p_t c_t] = 0 \tag{3.7}$$

The sufficient condition for the cash-in-advance constraint to bind is that the nominal interest rate be strictly positive

$$\frac{(1+r)u'(c_t)p_{t+1}}{u'(c_{t+1})p_t} - 1 > 0 \tag{3.8}$$

I will assume that cash-in-advance constraint is binding in the rest of the paper. The first order conditions have usual interpretations. For the optimal choice of consumption, the household equates the marginal benefit of spending one unit of money (the left hand side of (3.5)) with the marginal cost (the right hand side of (3.5)) which is the sum total of Langrangian multipliers associated with the cash-in-advance constraint and the law of motion of money holding.

(3.6) states that by not spending one unit of money in the current period, the household relaxes the cash-in-advance constraint and the constraint on the household money balance next period. (3.5) and (3.6) together imply that the shadow value of money balance ω_{Mt} is given by

$$\omega_{Mt} = \frac{1}{1+r}\frac{u'(c_{t+1})}{p_{t+1}} \tag{3.9}$$

3.5 The Optimal Choice of the Set of Wage Trajectories Offered $S_{PW}(t)$ and the Level of Vacancy v_t

The household instructs the firm to post the measure of vacancy v_t and the set of wage trajectories $S_{PW}(t)$ which maximize the total net return on vacancies. While posting vacancies and the associated wage trajectories, the household takes the aggregate level of vacancy $\hat{v}_t$, the aggregate probability distribution of marginal values offered $\hat{F}_t(\Omega_{et}(W_t))$, the aggregate probability distribution of marginal values earned $\hat{G}_t(\Omega_{et}(W_t))$, and the optimal strategies of other households as given. Let $\hat{\Omega}_{ut}$ be the household belief about the marginal value of unemployed workers. Then, the household will not post any wage trajectory which gives workers marginal value of less than $\hat{\Omega}_{ut}$ since it will not be able to attract any worker.

Let us first consider the choice of optimal set of wage trajectories offered $S_{PW}(t)$. The net expected return on a vacancy posted with a wage trajectory W_t is given by

$$R(W_t) \equiv -k + \frac{s(\hat{v}_t)}{\hat{v}_t}\Big[\hat{u}_t + (1-\hat{u}_t)\hat{G}_t(\Omega_{et}(W_t))\Big]\Omega_{Jt}(W_t) \tag{3.10}$$

The first term in the right hand side is the per-period cost of posting vacancy and the second term is the expected benefit on the vacancy with wage trajectory W_t. The expected benefit on a vacancy with wage trajectory W_t is the product of the marginal value of filled job at the wage trajectory W_t, $\Omega_{Jt}(W_t)$, and the matching probability of the vacancy with wage trajectory W_t. The matching probability of a vacancy with wage trajectory W_t is equal to the aggregate matching rate of vacancy times the proportion of workers in the economy with the marginal values less than and equal to $\Omega_{et}(W_t)$. It is immediately clear from (3.10) that by posting a wage trajectory with higher marginal values to workers, the household can increase the matching probability.

From the envelope condition, the marginal value of a filled job with wage trajectory W_t is given by

$$\Omega_{Jt}(W_t) = \frac{1}{1+r}\Big[(p_{t+1}y - w_{t+1})\omega_{M_{t+1}} +$$

$$\Big[1-\rho - s(\hat{v}_{t+1})(1-\hat{F}_{t+1}(\Omega_{et+1}(W_{t+1})))\Big]\Big]\Omega_{Jt+1}(W_{t+1}) \tag{3.11}$$

The first term in the right hand side is the flow value of profit evaluated using the marginal value of money balance at time $t+1$. The second term is the expected continuation value of the match. The second term reflects the fact that the match can get destroyed either due to idiosyncratic shocks or the employed worker leaving the match for a better job. The second term also shows that workers' turnover is lower at wage trajectories with higher marginal valuation.

The household will post wage trajectories such that

$$W_t \in \underset{W_t}{\text{argmax}}\ R(W_t) \equiv R^* \tag{3.12}$$

Let W_t^* be an optimal wage trajectory. Then the household will post wage trajectories other than W_t^* if and only if all the other posted wage trajectories give return equal to R^*. The total net return on all the vacancies for the household is given by

$$TR \equiv -kv_t + \int_{i\in S_{PW}(t)} \frac{s(\hat{v}_t)}{\hat{v}_t}\big[\hat{u}_t + (1-\hat{u}_t)\hat{G}_t(\Omega_{et}(i))\big]\Omega_{Jt}(i)v_t(i)di \tag{3.13}$$

Utilizing (3.12) we get

$$TR = R^* v_t \tag{3.14}$$

For the optimal choice of v_t, the household will maximize $v_t \in \text{argmax}_{v_t}\ TR$. The first order condition is

$$k = \frac{s(\hat{v}_t)}{\hat{v}_t}\big[\hat{u}_t + (1 - \hat{u}_t \hat{G}(\Omega_{et}(W_t^*))\big]\Omega_{Jt}(W_t^*) \tag{3.15}$$

(3.15) states that for the optimal choice of vacancy, the household equates the marginal cost of posting vacancy (k) with the marginal benefit of posting vacancy.

3.6 The Optimal Choice of the Reservation Wage Trajectories

An employed worker of the household accepts any wage trajectory with higher marginal values than what he derives at his present wage trajectory. An unemployed worker of the household accepts any wage trajectory with the marginal value at least equal to the marginal value of unemployed worker. From the envelope condition, the marginal value of an employed worker at wage trajectory W_t to the household, $\Omega_{et}(W_t)$, is given by

$$\Omega_{et}(W_t) =$$

$$\frac{1}{1+r}\Big[w_{t+1}\omega_{Mt+1} - (1+\phi) + \big[1 - \rho - s(\hat{v}_{t+1})(1 - \hat{F}_{t+1}(\Omega_{et+1}(W_{t+1})))\big]\Omega_{et+1}(W_{t+1})$$

$$+\rho\Omega_{ut+1} + s(\hat{v}_{t+1})(1 - \hat{F}_{t+1}(\Omega_{et+1}(W_{t+1}))) \int_{x|\Omega_{et+1}(x) > \Omega(W_{t+1})} \Omega_{et+1}(x) d\hat{F}_{t+1}(\Omega_{et+1}(x))\Big]$$

$$\forall\ w \tag{3.16}$$

(3.16) equates the opportunity cost of employment with the discounted expected benefit from the employment at the wage trajectory W_t. The expected benefit reflects the fact that the employed worker gets the wage w_{t+1} at time $t+1$ and incurs disutility from working and searching. Also at the end of the next period he can either remain employed at the same wage trajectory, become unemployed or match with a better job.

Similarly, the marginal value of an unemployed worker is given by

$$\Omega_{ut} = \frac{1}{1+r}\Big[-1 + (1 - s(\hat{v}_{t+1})(1 - \hat{F}(\Omega_{ut+1})))\Omega_{ut+1} +$$

$$s(\hat{v}_{t+1})(1 - F(\Omega_{ut+1})) \int_{x|\Omega_{et+1}(x) \geq \Omega_{ut+1}} \Omega_{et+1}(x) d\hat{F}_{t+1}(\Omega_{et+1}(x))\Big] \tag{3.17}$$

(3.17) has similar interpretation as (3.16). The term in the right hand side refers to the fact that an unemployed worker incurs disutility from searching and at the end of next period he either remains unemployed or becomes employed.

Given the optimal job acceptance strategy of unemployed workers, unemployed workers will accept any wage trajectory W_t such that

$$\Omega_{et}(W_t) \geq \Omega_{ut} \tag{3.18}$$

4. Equilibrium

I restrict attention to a particular class of equilibrium. Firstly, I require that cash in advance constraint is binding ($g > \frac{1}{1+r}$). Secondly, I require that the equilibrium be stationary in the sense that all real wage trajectories be constant i.e. nominal wages grow at the common rate of money creation g. Also the levels of consumption, unemployment, employment, and vacancy be constant. This presupposes that money is neutral.

Now define, real wage $w \equiv \frac{w_t}{p_t}$; the real money balance $M \equiv \frac{M_t}{p_t}$; the marginal value of real money balance $\Omega_M \equiv p_t \Omega_{Mt}$; and $\Omega_c \equiv p_t \omega_{ct}$. Given that the cash-in-advance constraint binds, we have $p_t = \frac{M_t}{c_t}$ and in the stationary equilibrium price will grow at the rate equal to the money creation rate i.e. $\frac{p_{t+1}}{p_t} = g$. Since in equilibrium all nominal wages grow at the same constant rate, the real wage trajectory associated with a nominal wage trajectory W_t is given by $\{\frac{w_t}{p_t}, \frac{w_{t+1}}{p_{t+1}}, ...\} \equiv \{w, w, ...\}$ and, therefore, in stationary environment $F(\Omega_{et}(W_t)) \equiv F(\Omega_e(w))$ and $G(\Omega_{et}(W_t)) \equiv G(\Omega_e(w))$.

Using above normalizations, the marginal value of an employed worker with the real wage w, $\Omega_e(w)$ can be written as (equation 3.16)

$$\Omega_e(w) = \frac{1}{r + \rho + s(\hat{v})(1 - F(\Omega_e(w)))} \Big[w\Omega_M - (1 + \phi) + \rho\Omega_U$$

$$+ s(\hat{v})(1 - F(\Omega_e(w))) \int_{x|\Omega_e(x) > \Omega_e(w)} \Omega_e(x) dF(\Omega_{(x)}) \Big] \quad \forall \; w \tag{4.1}$$

Proposition 1: The marginal value of employment at the real wage w, $\Omega_e(w)$, is strictly increasing in w.

The intuition of the result is follows. An employed worker with higher real wage trajectory gets higher current utility compared to an employed worker with a lower real wage trajectory. At the same time, next period the employed worker with a higher real wage trajectory faces the same aggregate matching rate as well as the aggregate real wage offer distribution as an employed worker with lower real wage trajectory. Thus, $\Omega_e(w)$ is strictly increasing in w.

Given the fact that $\Omega_e(w)$ is strictly increasing in the real wage w, the probability distributions of marginal values posted and earned become equivalent to the probability distributions of real wage offers and earnings respectively.

$$F(\Omega_e(w)) \equiv F(w) \ \& \ G(\Omega_e(w)) \equiv G(w) \tag{4.2}$$

Given proposition 1, from (3.18) it follows that the optimal real reservation wage $\underline{w}$ is given by $\Omega(\underline{w}) = \Omega_u$ which implies

$$\phi = \underline{w}\Omega_M \tag{4.3}$$

(4.3) equates the disutility of working with the utility of working at the real reservation wage $\underline{w}$. The implication is that if the marginal value of real money balance falls, the real reservation wage must rise in order to induce unemployed workers to work. Similarly, if the marginal value of real money balance rises, the real reservation wage falls.

4.1 Steady State Household Unemployment Rate and the Distribution of Real Wage Earnings $G(w)$

Given the optimal strategies of workers, one can easily derive steady-state unemployment rate and the real wage earnings distribution $G(w)$ of the representative household. (3.4) implies that the measure of unemployed workers in the household satisfies

$$u = (1-u)\rho + u(1 - s(\hat{v})(1 - \hat{F}(\underline{w}))) \tag{4.4}$$

which implies

$$u = \frac{\rho}{\rho + (1 - (1 - s(\hat{v})(1 - \hat{F}(\underline{w}))))} \tag{4.5}$$

From (3.3) we get

$$(1-u)G(w)[\rho + (1 - \hat{F}(w))s(\hat{v})] = us(\hat{v})[\hat{F}(w) - \hat{F}(\underline{w})] \tag{4.6}$$

(4.6) states that in the steady state, the outflow from the pool of workers earning real wage w and less should equal the inflow to the pool. The employed workers leave this pool either due to idiosyncratic shock or workers receiving real wage offers higher than w. The inflow to the pool comes from unemployed workers who receive the real wage offer of w and less. (4.5) and (4.6) together imply that the household distribution of real wage earnings $G(w)$ is given by

$$G(w) = \frac{\rho\hat{F}(w)}{\rho + (1 - \hat{F}(w))s(\hat{v})} \tag{4.7}$$

4.2 Equilibrium

Equilibrium: A stationary monetary equilibrium (SME) is defined as a collection of variables $\{c, M, v, \hat{v}, u, \hat{u}, \underline{w}, \hat{\underline{w}}\}$ and the distributions $\{F(w), G(w), \hat{F}(w), \hat{G}(w)\}$ such that

(i) Given $\hat{F}(w), \hat{G}(w)$ and $\hat{v}$, the household choice variables $\{c,\ M,\ v,\ F(w)\}$ solve (PH);

(ii) the real reservation wage of an unemployed worker $\underline{w}$ satisfies (4.3);

(iii) u is given by (4.5);

(iv) $G(w)$ is given by (4.7);

(v) the expected return on each vacancy is equal, $R(w) = R^* \ \forall \ w \in [\underline{w}, \overline{w}]$ where $\overline{w}$ is the highest real wage posted by the household;

(vi) aggregate variables are equal to the relevant household variables $\hat{F}(w) = F(w)$, $\hat{G}(w) = G(w)$, $\hat{v} = v$, $\hat{u} = u$, $\hat{\underline{w}} = \underline{w}$;

(vii) goods market clears $Jy = c$;

(viii) the marginal value of real money balance $0 < \Omega_M < \infty$.

The optimal choices of household and the equilibrium conditions induce following equilibrium relations.

The marginal value of real money balance is given by

$$\Omega_M = \frac{u'(c)}{(1+r)g} \tag{4.8}$$

In equilibrium, no firm will post real wage less than the reservation wage $\underline{w}$ since it will not attract any worker and thus $F(\underline{w}) = 0$. Then, unemployment rate u is given by

$$u = \frac{\rho}{\rho + s(v)} \tag{4.9}$$

The distribution of real wage earnings satisfies

$$G(w) = \frac{\rho F(w)}{\rho + (1 - F(w))s(v)} \tag{4.10}$$

Given that the cash-in-advance constraint binds, goods market clears and in equilibrium the measure of employed workers e equals the measure of employees of firms J, we have

$$M = Jy \equiv \frac{s(v)}{\rho + s(v)} y = c \tag{4.11}$$

The first equality is the quantity theory of money and the second is the market clearing condition.

The equilibrium condition that the expected return on each vacancy be equal and (3.15) imply that the equilibrium measure of vacancies v implicitly solves

$$k = \frac{s(v)}{v} \frac{\rho}{\rho + s(v)} \Omega_J(\underline{w}) \tag{4.12}$$

where $\Omega_J(\underline{w})$ is the marginal value of a job at the lowest wage offered. (4.12) equates the marginal cost of posting vacancy with the discounted expected marginal benefit from the job at the real reservation wage. After substituting for $\Omega_J(\underline{w})$ in (4.12) we get

$$k = \frac{s(v)}{v} \frac{\rho}{(\rho + s(v))} \frac{(y - \underline{w})\Omega_M}{(r + \rho + s(v))} \tag{4.13}$$

The equilibrium condition for the measure of vacancy is similar to the equilibrium condition derived by Mortensen (2000). The only difference is that the measure of vacancy in this model depends on the marginal value of real money balance which is endogenously determined in the economy. This is the consequence of embedding Mortensen model in the general equilibrium monetary model. Using (4.3), (4.8), and (4.11) we can eliminate $\underline{w}$ and Ω_M and get

$$kv = s(v) \frac{\rho}{(\rho + s(v))(r + \rho + s(v))} \left[\frac{u'\left(\frac{s(v)y}{\rho + s(v)}\right)}{(1+r)g} y - \phi \right] \tag{4.14}$$

It is clear from (4.14) that the solutions for the measure of vacancy v depend on the form of the household utility function. Also, notice that the largest value the measure of vacancy v can take in equilibrium is given by

$$\frac{u'\left(\frac{s(v)y}{\rho + s(v)}\right)}{(1+r)g} y = \phi \tag{4.15}$$

Denote the solution of (4.15) as $\overline{v}$ and the associated consumption as $\overline{c}$. The lowest value the equilibrium measure of vacancy v can take is zero and the associated consumption will also be zero. Now, define the coefficient of relative risk aversion as $\theta(c) \equiv -\frac{u''(c)}{u'(c)}c$. Then, the following proposition characterizes the possible solutions of v for different types of utility function.

Proposition 2:

(i) If the utility function is such that $\lim_{c \to 0} u'(c)c \to 0$, then (4.14) has a solution at $v = 0$. In addition, if $\lim_{c \to 0} \theta(c) < 1$ where $\theta(c)$ is the coefficient of relative risk-aversion, then (4.14) also has a solution at $v > 0$. Moreover, if the utility function is linear, then the solution at $v > 0$ is unique.

(ii) If the utility function is such that $\lim_{c \to 0} u'(c)c > 0$ and $\theta(c) \geq 1$, $\forall\ 0 \leq c \leq \overline{c}$, then (4.14) has a unique solution at $v > 0$.

In the rest of the paper, I will assume that there exists a unique equilibrium level of vacancy $v > 0$. The equilibrium at $v = 0$ corresponds to no-trade equilibrium. However, the equilibrium at $v = 0$ is unstable and the equilibrium at $v > 0$ is stable in the sense that the economy starting from any initial level of vacancy $v_0 > 0$ converges to the equilibrium $v > 0$.

The equilibrium condition that the expected net return on each vacancy be equal implies that, for a given $v > 0$, the distribution of real wage offers implicitly solves

$$\frac{y - w}{(\rho + s(v)(1 - F(w)))(r + \rho + s(v)(1 - F(w)))} = \frac{y - \underline{w}}{(\rho + s(v))(r + \rho + s(v))} \tag{4.16}$$

(4.16) is a quadratic equation in the distribution of real wage offers $F(w)$. The equilibrium $F(w)$ is the positive root of the quadratic equation and is given by

$$F(w) = \frac{r + 2(\rho + s(v))}{2s(v)}\left[1 - \sqrt{\frac{r^2 + 4(\rho + s(v))(r + \rho + s(v))\frac{y-w}{y-\underline{w}}}{(r + 2(\rho + s(v)))^2}}\right] \tag{4.17}$$

Putting $F(\overline{w}) = 1$ in (4.17), one can solve for the upper real wage support of the offer distribution $\overline{w}$. $\overline{w}$ is given by

$$\overline{w} = \underline{w} + \left[1 - \frac{\rho(r + \rho)}{(\rho + s(v))(r + \rho + s(v))}\right](y - \underline{w}) \tag{4.18}$$

The equations for the distribution of the real wage offers and its upper real wage support are identical in form to the ones derived in Moretensen (2000). However, the crucial difference is that in the current model unlike Mortensen (2000) model, the real reservation wage $\underline{w}$ and the measure of vacancy v depend on the endogenously determined marginal value of real money balance.

For equilibrium $v > 0$, the marginal value of real money Ω_M is strictly positive and well-defined. Hence, we have following proposition.

Proposition 3: There exists a unique SME with dispersed real wage offer and earnings distributions.

To further characterize the SME with dispersed real wages it is instructive to compare this equilibrium with the stationary competitive monetary equilibrium. In the competitive equilibrium case labor market is frictionless.

Proposition 4: In the SME with dispersed real wages, consumption, output, employment, and the real wages are lower compared to the competitive equilibrium case.[7]

Proposition 4 is the consequence of the fact that in the SME with dispersed real wages, firms enjoy monopsony power in the labor market. Because of that firms make strictly positive profit on filled jobs. On the other hand, in the competitive equilibrium firms make zero profit on filled jobs. Because of this, employment level and hence output and consumption in the SME with dispersed real wages is lower compared to the competitive equilibrium case. Also, in the competitive case real wage is equal to the marginal product of workers. But, in the SME with dispersed real wages even the highest real wage paid is lower than the marginal product of workers.

[7] The competitive equilibrium program has two solutions depending on whether the work-force constraint binds or not. If the work-force constraint binds, optimal $e = 1$. In the case, it does not bind, optimal $e < 1$ (see the proof of proposition 4 in appendix).

It is also important to note that in the current environment for the stationary monetary equilibrium to exist search by employed workers is necessary. In the absence of on-the-job search, monetary equilibrium cannot exist.

Proposition 5: There does not exist any stationary monetary equilibrium when only unemployed workers search.

The intuition for this result is straight-forward. In this case as shown by Diamond (1971), the real wage offer distribution will be degenerate at the real reservation wage. At this real wage, the expected capital gain to the unemployed workers is zero. In the case where search has strictly positive disutility cost, unemployed workers would prefer not to participate in the labor market and thus there will be no employment, output, and trade.

In the next section, I compare the SME with dispersed real wages with the social optimum which further highlights the characteristics of the SME with dispersed real wages.

5. The Optimal Inflation

The social planner maximizes

$$\max_{c_t, v_t, u_{t+1}} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \Big[u(c_t) - (1+\phi)(1-u_t) - u_t - kv_t \Big] \tag{5.1}$$

subject to

$$u_{t+1} = \rho(1-u_t) + (1 - s(v_t))u_t \quad \forall\ t \tag{5.2}$$

$$c_t = y(1-u_t) \tag{5.3}$$

The first constraint is the labor market constraint which tells that the next period level of unemployment equals the sum of the measure of employed workers becoming unemployed in the current period and the measure of unemployed workers of the current period unable to get job. The second constraint is the goods market clearing constraint.

Let λ_{ut} and λ_{ct} be the Langrangian multipliers associated with (5.2) and (5.3) respectively. Then, the first order conditions for the social optimum are given by

$$c_t: \ u'(c_t) = \lambda_{ct} \tag{5.4}$$

$$v_t: \ k = -\lambda_{ut} s'(v_t) u_t \tag{5.5}$$

$$u_{t+1}: \ \lambda_{ut} = \frac{1}{1+r} \Big[\phi - y\lambda_{ct+1} + [1 - \rho - s(v_{t+1})]\lambda_{ut+1} \Big] \tag{5.6}$$

Combining (5.4), (5.5), and (5.6) we get

$$\frac{k}{s'(v_t)u_t} = \frac{1}{1+r}\Big[u'(c_{t+1})y - \phi + \frac{k(1-\rho-s(v_{t+1}))}{s'(v_{t+1})u_{t+1}}\Big] \tag{5.7}$$

In the stationary state, (5.7) reduces to

$$k = \frac{\rho s'(v)}{(\rho+s(v))(r+\rho+s(v)}[u'(c)y-\phi] \tag{5.8}$$

It can be easily shown that (5.8) has unique solution. Let the elasticity of matching function with respect to vacancy be $\eta \equiv \frac{s'(v)v}{s(v)}$. The following proposition compares the social optimal level of vacancy with the market equilibrium level of vacancy.

Proposition 6: The Friedman rule ($g = \frac{1}{1+r}$) is not optimal. When monetary growth rate follows Friedman rule, firms create too many vacancies and unemployment rate is too low. The optimal rate of inflation rate, g^*, is given by

$$g^* = \frac{1}{(1+r)[\eta + \frac{(1-\eta)\phi}{u'(c)y}]} \tag{5.9a}$$

$$\lim_{\eta\to 1} g* = \frac{1}{1+r} \tag{5.9b}$$

The result that the Friedman rule is not optimal is not surprising. The economy has two sources of inefficiency- the binding cash-in-advance constraint and the monopsony power of firms in the labor market. The Friedman rule removes the first source of inefficiency but not the second. Because of monopsony power, when firms face zero cost of real money balance they set too low real wages and create too many vacancies. In such case, inflation rate exceeding the Friedman rule is going to be optimal.

Notice that when the elasticity of matching function with respect to vacancy η approaches unity, the optimal inflation rate approaches the Friedman rule. This happens because when $\eta = 1$, the economy satisfies Hosios condition i.e. the elasticity of matching function with respect to vacancy equals the bargaining power of firms. In this case, all the externalities generated by search friction is internalized. With the Hosios condition satisfied, the only source of sub-optimality in the economy is the binding cash-in-advance constraint which is removed by the Friedman rule. However, given the assumptions about the matching function $0 < \eta < 1$.

6. Inflation

In this section, I analyze the effects of changes in the inflation rate on equilibrium variables particularly on the distributions of real wage offers and earnings.

6.1 Effects of Inflation in the SME with dispersed real wages

Proposition 7: An increase in the inflation rate in the SME with dispersed real wages reduces the level of vacancy, consumption, and output and increases the unemployment rate.

In the SME with dispersed real wages, an increase in the inflation rate reduces consumption, output, and employment. This happens because an increase in the inflation rate reduces the expected benefit from working which increases the real reservation wage of unemployed workers for a given consumption level. This induces firms to reduce the equilibrium level of vacancy which leads to lower output and consumption and higher unemployment rate.

The effect of inflation in the SME with dispersed real wages is similar to that in the standard cash-in-advance economies (e.g. Cooley and Hansen 1990) where an increase in the inflation rate induces households to shift from consumption good which requires cash to leisure which does not require cash.[8]

Proposition 8: An increase in the inflation rate increases the real reservation wage and has ambiguous effect on the highest real wage offered if the marginal value of real money balance falls. In the case, an increase in the inflation rate increases the marginal value of real money balance, the reservation wage and the highest real wage offered fall.

An increase in the inflation rate directly reduces the marginal value of real money balance Ω_M. But, through its indirect effect on consumption which falls, it may increases Ω_M. Thus, the overall effect of the inflation rate on Ω_M is ambiguous. A reduction in Ω_M reduces the expected benefit from working (eq. 4.3) which implies that for a given disutility cost of working the real reservation wage $\underline{w}$ must rise in order to induce unemployed workers to work. For the same reason, an increase in Ω_M leads to a fall in $\underline{w}$.

An increase in the inflation rate accompanied by an increase in the real reservation wage $\underline{w}$ has an ambiguous effect on the highest real wages offered $\overline{w}$. An increase in $\underline{w}$ reduces the monopsony power of firms and directly increases $\overline{w}$. But, an increase in the inflation rate by reducing the level of vacancy and thus the matching rate of workers also increases the monopsony power of firms which may prevent $\overline{w}$ from rising. In the case, an increase in the inflation rate reduces $\underline{w}$ then $\overline{w}$ unambiguously falls. A fall in $\underline{w}$ and the level of vacancy reinforces the monopsony power of firms.

Proposition 9: An increase in the inflation rate reduces the range of real wage offers and earnings if the marginal value of money falls. In the case, an increase in the inflation rate increases the marginal value of real money balance, the effect of inflation rate on the range of real wage offers and earnings is ambiguous.

Inflation rate affects the range of real wages through its effect on the level of vacancy and the real reservation wage. Other things remaining the same, a fall in the level of

[8] In the competitive case with interior solution, one can easily show that inflation reduces employment and consumption.

vacancy by reducing the highest real wage reduces the range of real wages. On the other hand, for a given level of vacancy, an increase in the real reservation wage reduces the range of real wages and vice versa. This happens because of the equilibrium condition that the net expected return on each vacancy is equal.

A filled job paying real reservation wage faces a higher effective discount rate $(r+\rho+s(v))$ than a filled job paying the highest real wage posted $(r+\rho)$. The result is that a change in the highest real wage offered for a given change in the real reservation wage has to be smaller in order to satisfy the equilibrium condition of equal net expected return on each vacancy.

Since, inflation rate reduces the level of vacancy, in the case inflation rate increases the real reservation wage the range of real wages falls unambiguously. On the other hand, in the case inflation rate reduces the real reservation wage, the effect of inflation rate on the range of real wages is ambiguous.

Proposition 10:

(i) An increase in the inflation rate has ambiguous effect on the distribution of real wage offers $F(w)$ if the real reservation wage $\underline{w}$ rises. If the real reservation wage $\underline{w}$ falls, then for small discount rate r the proportion of low real wage job offers rises.

(ii) An increase in the inflation rate has ambiguous effect on the distribution of real wage earnings $G(w)$ if there is stochastic improvement in the distribution of real wage offers $F(w)$. If there is no stochastic improvement in the distribution of real wage offers $F(w)$, then the proportion of low real wage earnings increases.

Inflation rate affects the distribution of real wage offers through its effect on the real reservation wage and the level of vacancy. An increase in the inflation rate if it increases the real reservation wage reduces the monopsony power of firms and leads to stochastic improvement in the distribution of real wage offers. But, an increase in the inflation rate also reduces the level of vacancy which by increasing the monopsony power of firms may prevent any stochastic improvement in the distribution of real wage offers. In the case the real reservation wage falls, the monopsony power of firms increases unambiguously and firms post greater proportion of low wage jobs.

The effect of inflation rate on the distribution of real wage earnings depends on its effect on the distribution of real wage offers and the level of vacancy. An increase in the inflation rate reduces the level of vacancy which by reducing the matching rate of workers lowers the rate of movement of employed workers from low real wage jobs to high real wage jobs. This shifts the mass of employed workers from high real wage jobs to low real wage jobs. As discussed above, inflation rate has ambiguous effect on the distribution of real wage offers. If there is stochastic improvement in the distribution of the real wage offers, then it may prevent the shift of mass of employed workers from high real wage jobs to low real wage jobs caused by a fall in the level of vacancy. But, if there is no stochastic improvement in the distribution of real wage offers, then the proportion of employed workers with low real wage jobs unambiguously rises.

6.2 An Example

Let utility function $u(c) = lnc$, the matching function $s(v) = v^{0.98}$, $y = 1$, $k = 0.045$, $\rho = 0.064$, and $\phi = 0.25$. Let the time period be one quarter. Also set the discount rate $r = 0$. This assumption simplifies the welfare comparison. With $r = 0$, one has to just compare one period household utility for different inflation rates and not the entire path of the utilities. For these functional forms and parameter values, consider economies with gross inflation rate ranging from 0 percent to 500 percent (i.e. $g \in [1.0001, 1.5]$). The effects of changes in the inflation rate are depicted in figure 1 through figure 5.

Figure 1 depicts the effect of inflation rate on consumption. As expected, with the increase in the inflation rate consumption falls which is due to a fall in the level of vacancy v. However, a fall in the level of consumption does not necessarily lead to a decline in the household utility. Figure 2 shows that the household utility is maximized at a strictly positive level of inflation rate ($g = 1.0195$). As discussed earlier, because of monopsony power of firms the Friedman rule in this environment is not optimal. At low inflation rate, firms pay too low real wages and create too many vacancies. Inflation rate by increasing the real reservation wage erodes the monopsony power of firms.

Figure 3 depicts the effect of inflation rate on the real reservation wage and the highest real wage offered. With an increase in the inflation rate, the real reservation wage increases which is the consequence of the decline in the marginal value of real money balance. More interestingly, inflation has very small effect on the highest real wages offered. The positive effect of an increase in the real reservation wage is nullified by the negative effect of a fall in the level of vacancy which increases the monopsony power of firms. As expected, the range of real wages falls.

Figure 4 depicts the effect of inflation rate on the average real wage offer and earnings. Figure 4 shows that an increase in inflation rate leads to an increase in the average real wage offer but decline in the average real wage earnings. An increase in inflation rate may have positive or negative effect on the average real wage earnings. As discussed earlier, this happens because the effect of inflation rate on the distribution of real wage earnings depends on its effect on the the distribution of real wage offers and the level of vacancy. The positive effect of an increase in the average real wage offer on the average real wage earnings may be offset by a decline in the level of vacancy which reduces the rate of movement of employed workers from lower real wage jobs to higher real wage jobs.

Figure 5 shows the effect of inflation rates on the coefficient of variation of real wage offers and earnings. The variability of both the real wage offers and earnings falls with inflation. In the case of real wage offers, the coefficient of variation falls both because of a fall in the range and increase in the average real wage offer. In the case of real wage earnings, the coefficient of variation falls mainly because of the fall in the range of real wage earnings.

7. Conclusion

The paper has analyzed the effects of inflation rate on the distributions of within-skill real wage offers and real wage earnings in a general equilibrium monetary model. The paper finds that an increase in the inflation rate leads to higher real reservation wage, higher average real wage offer, and lower range of real wages. An increase in the inflation rate also reduces variability in both the real wage offers and earnings. The effect of inflation rate on average real wage earnings is ambiguous. The paper also finds that an increase in the inflation rate reduces the levels of vacancy and consumption and increases unemployment rate. The Friedman rule is not optimal. Because of the monopsony power of the firms in the labor market, the optimal inflation rate exceeds the discount rate.

Appendix

Proof of Proposition 1:

We prove this proposition by contradiction. $\Omega_e(w)$ is given by

$$\Omega_e(w) = \frac{1}{r+\rho}\Big[w\Omega_M - (1+\phi) + \rho\Omega_U$$

$$+s(\hat{v})(1-F(\Omega_e(w)))\big[\int_{x|\Omega_e(x)>\Omega_e(w)} \Omega_e(x)dF(\Omega_(x)) - \Omega_e(w)\big]\Big] \quad (A.1)$$

First, suppose that $\Omega_e(w)$ is constant in w which implies that if $w' > w$ then $\Omega_e(w') = \Omega_e(w)$. But, the right hand side of (A.1) is strictly higher with w' compared to that with w which contradicts our assumption.

Now suppose that $\Omega_e(w)$ is strictly decreasing in w i.e. if $w' > w$ then $\Omega_e(w') < \Omega_e(w)$. Again, we can see that the right hand side of (A.1) is strictly greater with w' compared to that with w which contradicts our assumption.

Proof of proposition 2:

The equilibrium value of vacancy is given by

$$kv = \frac{\rho}{r+\rho+s(v)}\big[\frac{u'(c)c}{(1+r)g} - \frac{\phi}{y}c\big] \equiv T(v) \quad (A.2)$$

From the above, it is clear that when $u'(c)c = 0$ for $c = 0$, then (A.2) has solution $v = 0$. But, when $u'(c)c > 0$ for $c = 0$, then (A.2) does not have solution at $v = 0$. Also, in this case $T(v) > kv$. From, (A.2) it is also clear that the maximum value which c or v can take is given by

$$\frac{u'(c)}{(1+r)g} = \frac{\phi}{y} \quad (A.3)$$

Denote solution of (A.3) by $\bar{c}$. It is also clear that at $\bar{c}$, $kv > T(v)$.

Differentiating $T(v)$ w.r.t. v, we get

$$T'(v) = \frac{\rho}{(r+\rho+s(v))^2}\Big[\frac{(r+\rho+s(v))y\rho}{(\rho+s(v))^2}\big[\frac{u''(c)c+u'(c)}{(1+r)g} - \frac{\phi}{y}\big]$$

$$-\big[\frac{u'(c)c}{(1+r)g} - \frac{\phi}{y}c\big]\Big]s'(v) \quad (A.4)$$

The coefficient of relative risk aversion is given by $-\frac{u''(c)c}{u'(c)} \equiv \theta(c)$. Then, (A.4) can written as

$$T'(v) = \frac{\rho}{(r+\rho+s(v))^2}\Big[\frac{(r+\rho+s(v))y\rho}{(\rho+s(v))^2}\big[\frac{(1-\theta(c))}{(1+r)g} - \frac{\phi}{y}\big]$$

$$-\big[\frac{u'(c)c}{(1+r)g} - \frac{\phi}{y}c\big]\Big]s'(v) \qquad (A.5)$$

From (A.5) it is clear that in the case $\theta(c) \geq 1$, $\forall\ 0 \leq c \leq \overline{c}$, $T'(v) < 0$. Thus, in the case $u'(c)c > 0$ when $c = 0$, there exists a unique solution.

(A.5) also shows that when $\lim_{c \to 0} \theta(c) < 1$ then $T'(v) \to \infty$ when $c \to 0$ since $\lim_{v \to 0} s'(v) \to \infty$. Thus, in the case $u'(c)c = 0$ when $c = 0$ there exists at least one non-trivial solution. In the linear utility case, one can also show that $T''(v) < 0$ which implies that in this case there will be a unique non-trivial equilibrium.

Proof of Proposition 4:

In the competitive case, the household maximization problem is

$$\max_{c_t, M_{t+1}, e_t, J_t} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t}\Big[u(c_t) - \phi e_t\Big]$$

subject to the cash-in-advance constraint given in (2.2), the law of motion of household nominal money balance,

$$M_{t+1} \leq M_t + (g-1)\hat{M}_t - p_t c_t + [p_t y - w_t]J_t + e_t w_t \qquad (A.6)$$

and the work-force constraint

$$e_t \leq 1 \qquad (A.7)$$

Let μ_{ct}, μ_{Mt}, and μ_{et} be the Langrangian multipliers associated with the cash-in-advance constraint and the constraints (A.6) and (A.7) respectively. Then, the first order conditions associated with the optimal choices are given by

$$c_t : \frac{u'(c_t)}{p_t} = \mu_{ct} + \mu_{Mt} \qquad (A.8)$$

$$M_{t+1} : \frac{1}{1+r}[\mu_{Mt+1} + \mu_{ct+1}] = \mu_{Mt} \qquad (A.9)$$

$$e_t : \phi \leq w_t \mu_{Mt} \qquad (A.10)$$

$$J_t : p_t y = w_t \qquad (A.11)$$

The slackness condition associated with (A.10) is given by

$$\mu_{et}[e_t - 1] = 0 \tag{A.11}$$

If constraint (A.7) is binding then $e_t = 1$ and when it is not binding there is unique interior solution with optimal $e_t < 1$.

Steady State Equilibrium

In the steady state

$$\mu_M = p_t \mu_{Mt} = \frac{u'(c)}{(1+r)g} \tag{A.12}$$

Case (i): Constraint (A.7) is binding.

In this case, $e = J = 1$, $c = y$, and $w = y$.

Case (ii): Constraint (A.7) is not binding.

In this case, $e = J$, $c = ey$, $w = y$ and the equilibrium e is given by

$$\phi = \frac{yu'(ey)}{(1+r)g} \tag{A.13}$$

Now, notice that in the SME with dispersed real wages because of friction in the labor market

$$\phi < \frac{yu'(ey)}{(1+r)g} \tag{A.14}$$

Given that the utility function is strictly increasing, it immediately follows that in the SME with dispersed real wages employment and hence consumption and output levels are lower compared to the competitive case. Also, in competitive equilibrium $w = y$, while in the SME with the dispersed real wages $\overline{w} < y$.

Notice also that in the competitive case with interior solution, the Friedman rule maximizes the social welfare. For the social optimal one requires that

$$yu'(c) = \phi \tag{A.15}$$

Comparing (A.13) with (A.15) we can immediately see that Friedman rule maximizes social welfare.

Proof of Proposition 5:

In the absence of on the job search, the return on a vacancy with real wage w is given by

$$R(w) = -k + \frac{s(\hat{v})}{\hat{v}} \frac{\hat{u}}{r+\rho}(y-w)\Omega_M \tag{A.16}$$

Clearly this is maximized at the lowest real wage firms can post which is equal to the reservation wage $\underline{w}$ of the unemployed workers. All the households will post only one real wage equal to $\underline{w}$. At this wage the marginal value of an unemployed worker is given by

$$\Omega_u = \frac{1}{r}[-1 + s(\hat{v})[\Omega_e(\underline{w}) - \Omega_u]] \tag{A.17}$$

Since, $\Omega_e(\underline{w}) = \Omega_u$ (A.16) implies

$$\Omega_u = -\frac{1}{r} \tag{A.18}$$

Since in the case unemployed worker does not participate in the labor market $\Omega_u = 0$, unemployed worker will be better off by not participating.

Proof of Proposition 6

The market equilibrium level of vacancy is given by

$$kv = s(v)\frac{\rho}{(\rho + s(v))(r + \rho + s(v))}\Big[\frac{u'\big(\frac{s(v)y}{\rho+s(v)}\big)}{(1+r)g}y - \phi\Big] \tag{A.19}$$

The social optimum level of vacancy is given by

$$k = \frac{\rho s'(v)}{(\rho + s(v))(r + \rho + s(v)}[u'(c)y - \phi] \tag{A.20}$$

Now, define the elasticity of matching function w.r.t. vacancy $\eta \equiv \frac{s'(v)v}{s(v)}$. (A.19) and (A.20) imply that the market and social levels of vacancy will be equal at the inflation rate g satisfying

$$\eta[u'(c)y - \phi] = [\frac{u'(c)y}{(1+r)g} - \phi] \tag{A.21}$$

Simplifying we get the optimal rate of g

$$g = \frac{1}{(1+r)[\eta + \frac{(1-\eta)\phi}{u'(c)y}]} \tag{A.22}$$

From (A.22) it is clear that the Friedman rule is optimal only when $\eta = 1$. But, given the restrictions on the matching function $0 < \eta < 1$. Also in equilibrium, $u'(c)y > \phi$ which implies that

$$g > \frac{1}{1+r} \tag{A.23}$$

Proof of Proposition 7:

The equilibrium level of vacancy is given by

$$kv = \frac{\rho}{r+\rho+s(v)}\left[\frac{u'(c)c}{(1+r)g} - \frac{\phi}{y}c\right] \equiv T(v) \tag{A.24}$$

We can immediately see that

$$\frac{dT(v)}{dg} < 0 \tag{A.25}$$

From (A.25) it follows that when equilibrium is unique, an increase in money creation rate reduces the equilibrium level of vacancy and thus employment, consumption, and output.

Proof of Proposition 8

The marginal value of real money balance is given by

$$\Omega_M = \frac{u'(c)}{(1+r)g} \tag{A.26}$$

Differentiating (A.26) w.r.t g we get

$$\frac{d\Omega_M}{dg} = \frac{1}{(1+r)g^2}\left[gu''(c)\frac{dc}{dg} - u'(c)\right] \tag{A.27}$$

Since $\frac{dc}{dg} < 0$, then for $u''(c) < 0$ $\frac{d\Omega_M}{dg}$ is of ambiguous sign.
The real reservation wage $\underline{w}$ is given by

$$\underline{w} = \frac{\phi}{\Omega_M} \tag{A.28}$$

Differentiating w.r.t. g we get

$$\frac{d\underline{w}}{dg} = -\frac{\phi}{\Omega_M^2}\frac{d\Omega_M}{dg} \tag{A.29}$$

From (A.29) it is clear that $\frac{d\underline{w}}{dg} > 0$, if $\frac{d\Omega_M}{dg} < 0$ and $\frac{d\underline{w}}{dg} < 0$, if $d\Omega_M > 0$.
The upper support of real wage distributions is given by

$$\overline{w} = y - \frac{\rho(r+\rho)}{(\rho+s(v))(r+\rho+s(v))}(y-\underline{w}) \tag{A.30}$$

Differentiating (A.30) w.r.t. g we get

$$\frac{d\overline{w}}{dg} = \frac{\rho(r+\rho)}{(\rho+s(v))(r+\rho+s(v))}\frac{d\underline{w}}{dg}$$

$$+\frac{\rho(r+\rho)}{(\rho+s(v))^2(r+\rho+s(v))^2}(r+2\rho+2s(v))(y-\underline{w})s'(v)\frac{dv}{dg} \tag{A.31}$$

We know $\frac{dv}{dg} < 0$. Thus, if $\frac{d\underline{w}}{dg} < 0$, then $\frac{d\overline{w}}{dg} < 0$. But, if $\frac{d\underline{w}}{dg} > 0$, then $\frac{d\overline{w}}{dg}$ is of ambiguous sign.

Proof of Proposition 9

The range of real wages offered and earned is given by

$$\overline{w} - \underline{w} = \Big[1 - \frac{\rho(r+\rho)}{(\rho + s(v))(r+\rho+s(v))}\Big](y - \underline{w}) \tag{A.32}$$

Differentiating (A.32) w.r.t. g we get

$$\frac{d(\overline{w}-\underline{w})}{dg} = -\Big[1 - \frac{\rho(r+\rho)}{(\rho+s(v))(r+\rho+s(v))}\Big]\frac{d\underline{w}}{dg}$$

$$+\frac{\rho(r+\rho)}{(\rho+s(v))^2(r+\rho+s(v))^2}(y-\underline{w})s'(v)\frac{dv}{dg} \tag{A.33}$$

Given that $\frac{dv}{dg} < 0$, if $\frac{d\underline{w}}{dg} > 0$, then $\frac{d(\overline{w}-\underline{w})}{dg} < 0$. If $\frac{d\underline{w}}{dg} < 0$, then $\frac{d(\overline{w}-\underline{w})}{dg}$ is of ambiguous sign.

Proof of Proposition 10

Proposition 10(i)

The wage offer distribution $F(w)$ is given by

$$F(w) = \frac{r+2(\rho+s(v))}{2s(v)}\Big[1 - \sqrt{\frac{r^2 + 4(\rho+s(v))(r+\rho+s(v))\frac{y-w}{y-\underline{w}}}{(r+2(\rho+s(v)))^2}}\Big] \tag{A.34}$$

Let $L \equiv (r+2\rho+s(v))^2$. Then

$$\frac{dl}{dg} = 4(r+2\rho+2s(v))s'(v)\frac{dv}{dg} \tag{A.35}$$

Let $H \equiv r^2 + 4(r+\rho+s(v))(\rho+s(v))\frac{y-w}{y-\underline{w}}$. Then,

$$\frac{dH}{dg} = \frac{4(y-w)}{(y-\underline{w})^2}\Big[(y-\underline{w})(r+2\rho+2s(v))s'(v)\frac{dv}{dg} + (\rho+s(v))(r+\rho+s(v))\frac{d\underline{w}}{dg}\Big] \tag{A.36}$$

Differentiating (A.35) w.r.t. g we get

$$\frac{dF(w)}{dg} = -[1 - \sqrt{\frac{H}{L}}]\frac{\rho}{s(v)^2}\frac{dv}{dg}$$

$$-(\frac{H}{L})^{-1/2}\frac{r+2(\rho+s(v))}{4s(v)L^2}\frac{4L(y-w)}{(y-\underline{w})^2}\Big[(y-\underline{w})(r+2\rho+2s(v))s'(v)\frac{dv}{dg}$$

$$+(r+\rho+s(v))(\rho+s(v))\frac{d\underline{w}}{dg}\Big]$$

$$+(\frac{H}{L})^{-1/2}\frac{r+2(\rho+s(v))}{s(v)L^2}\Big[r^2+4(\rho+s(v))(r+\rho+s(v))(r+2\rho+2s(v))s'(v)\frac{y-w}{y-\underline{w}}\frac{dv}{dg}\Big] \tag{A.37}$$

Rearranging and simplifying (A.37) we get

$$\frac{dF(w)}{dg} = -[1-\sqrt{\frac{H}{L}}]\frac{\rho}{s(v)^2}\frac{dv}{dg}$$

$$(y-w)(\rho+s(v))(r+\rho+s(v))\frac{r+2(\rho+s(v))}{Ls(v)(y-\underline{w})^2}\frac{H}{L}^{-1/2}\frac{d\underline{w}}{dg}$$

$$-(\frac{H}{L})^{-1/2}\frac{r+2(\rho+s(v))}{s(v)L^2}\Big[\frac{L(y-w)}{y-\underline{w}}-\big(r^2+4(\rho+s(v))(r+\rho+s(v))\frac{y-w}{y-\underline{w}}\big)\Big]s'(v)\frac{dv}{dg} \tag{A.38}$$

Now $L\equiv r^2+4(\rho+s(v))^2+4r(\rho+s(v))$. This implies that

$$\frac{L(y-w)}{y-\underline{w}}-\big(r^2+4(\rho+s(v))(r+\rho+s(v))\frac{y-w}{y-\underline{w}}\big)=r^2\big(\frac{y-w}{y-\underline{w}}-1\big) \tag{A.39}$$

Putting (A.39) into (A.38) we get

$$\frac{dF(w)}{dg} = -[1-\sqrt{\frac{H}{L}}]\frac{\rho}{s(v)^2}\frac{dv}{dg}$$

$$(y-w)(\rho+s(v))(r+\rho+s(v))\frac{r+2(\rho+s(v))}{Ls(v)(y-\underline{w})^2}\frac{H}{L}^{-1/2}\frac{d\underline{w}}{dg}$$

$$-(\frac{H}{L})^{-1/2}\frac{r+2(\rho+s(v))}{s(v)L^2}r^2\big(\frac{y-w}{y-\underline{w}}-1\big)s'(v)\frac{dv}{dg} \tag{A.40}$$

If we set $r^2=0$, the proposition follows.

Proposition 10(ii)

The wage earnings distribution $G(w)$ is given by

$$G(w)=\frac{\rho F(w)}{\rho+s(v)(1-F(w))} \tag{A.41}$$

Differentiating (A.41) w.r.t. g we get

$$\frac{dG(w)}{dg} = \frac{\rho}{(\rho + s(v)(1 - F(w)))^2} \left[(\rho + s(v)) \frac{dF(w)}{dg} - F(w)(1 - F(w)) s'(v) \frac{dv}{dg} \right] \quad (A.42)$$

The proposition follows from (A.42).

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Figure 1
Effect of Inflation on Consumption

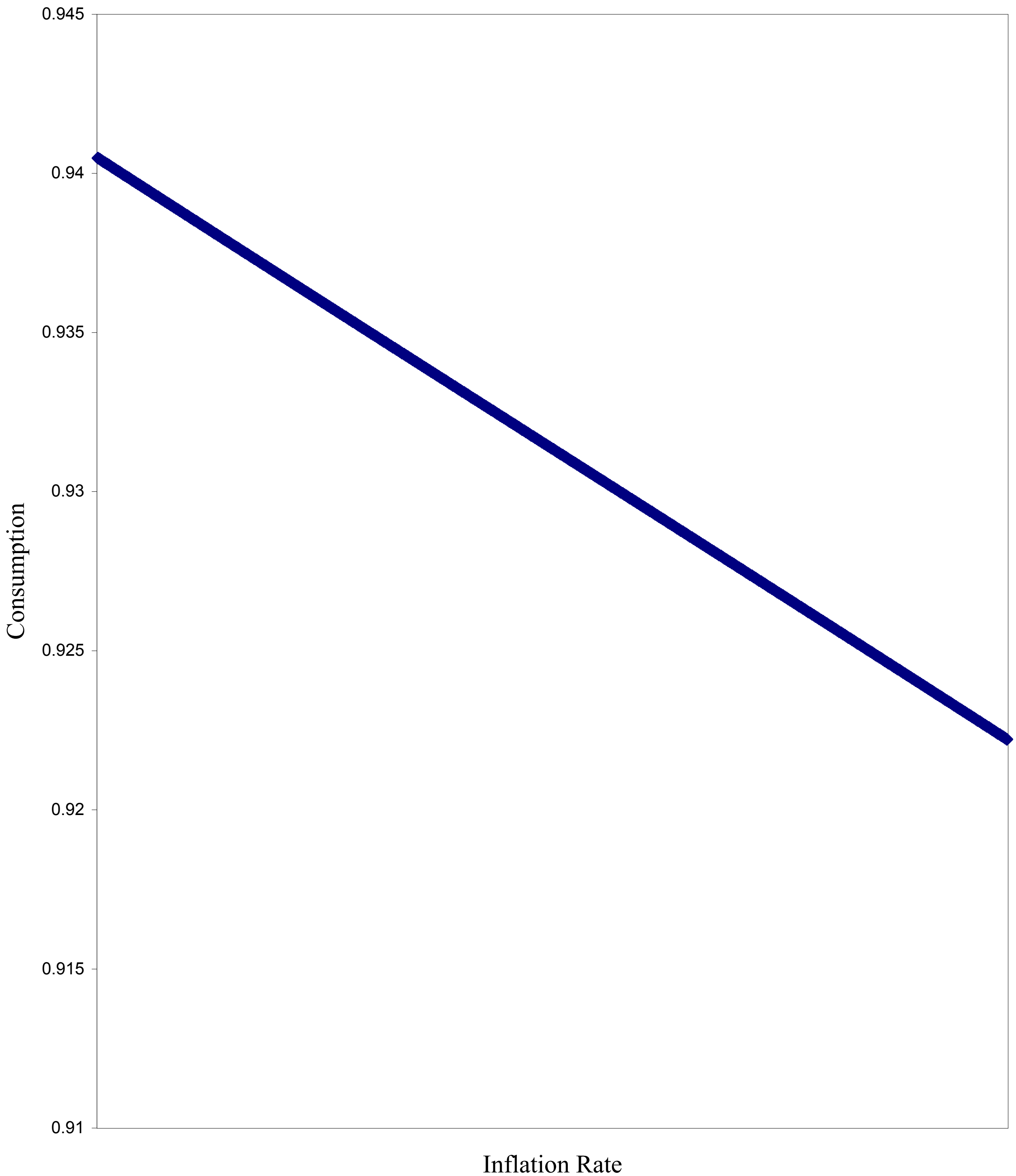

Figure 2
Effect of Inflation on Utility

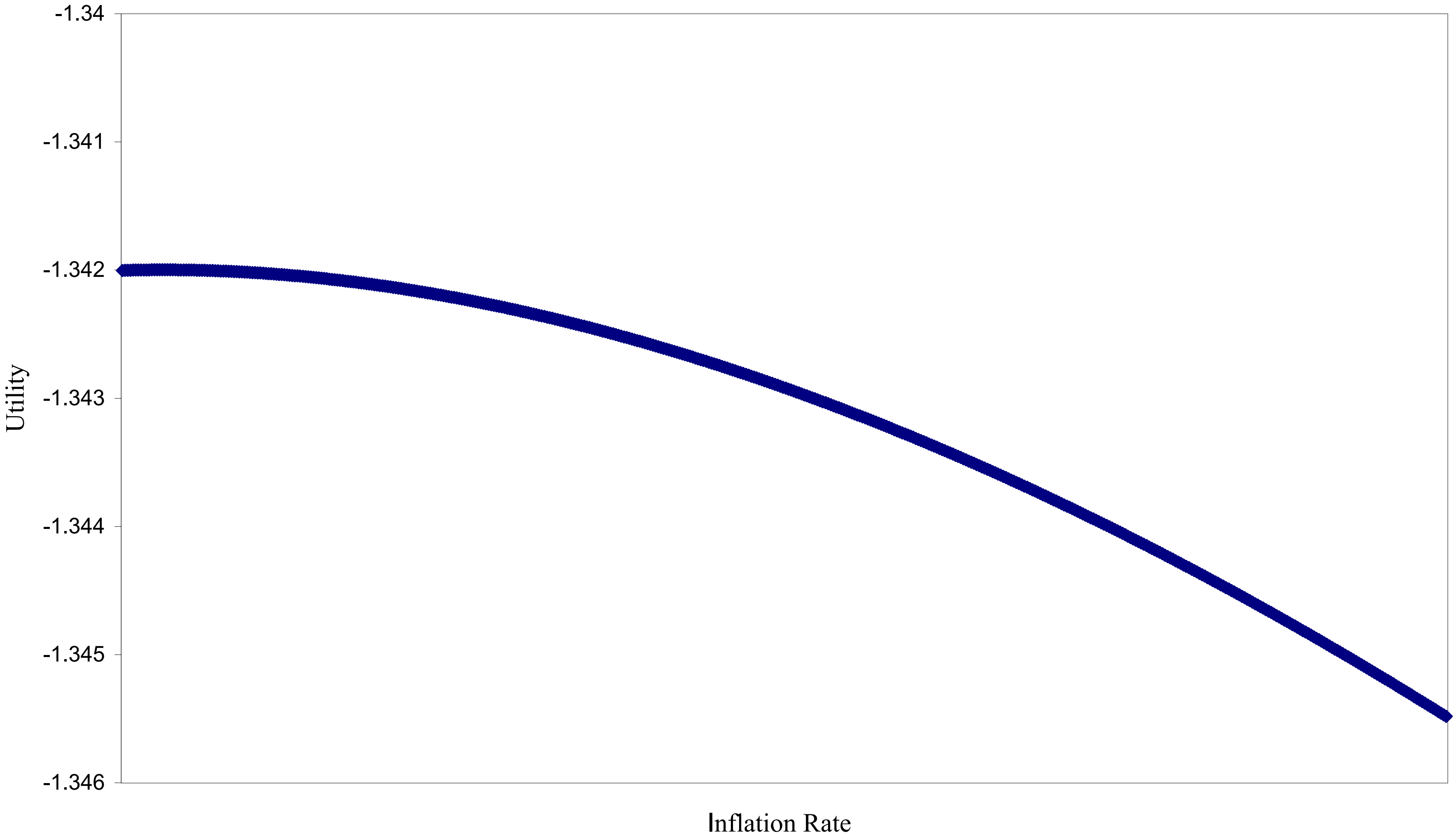

Figure 3
Effect of Inflation on the Upper and Lower Supports of the Real Wage Offer/Earnings Distribution

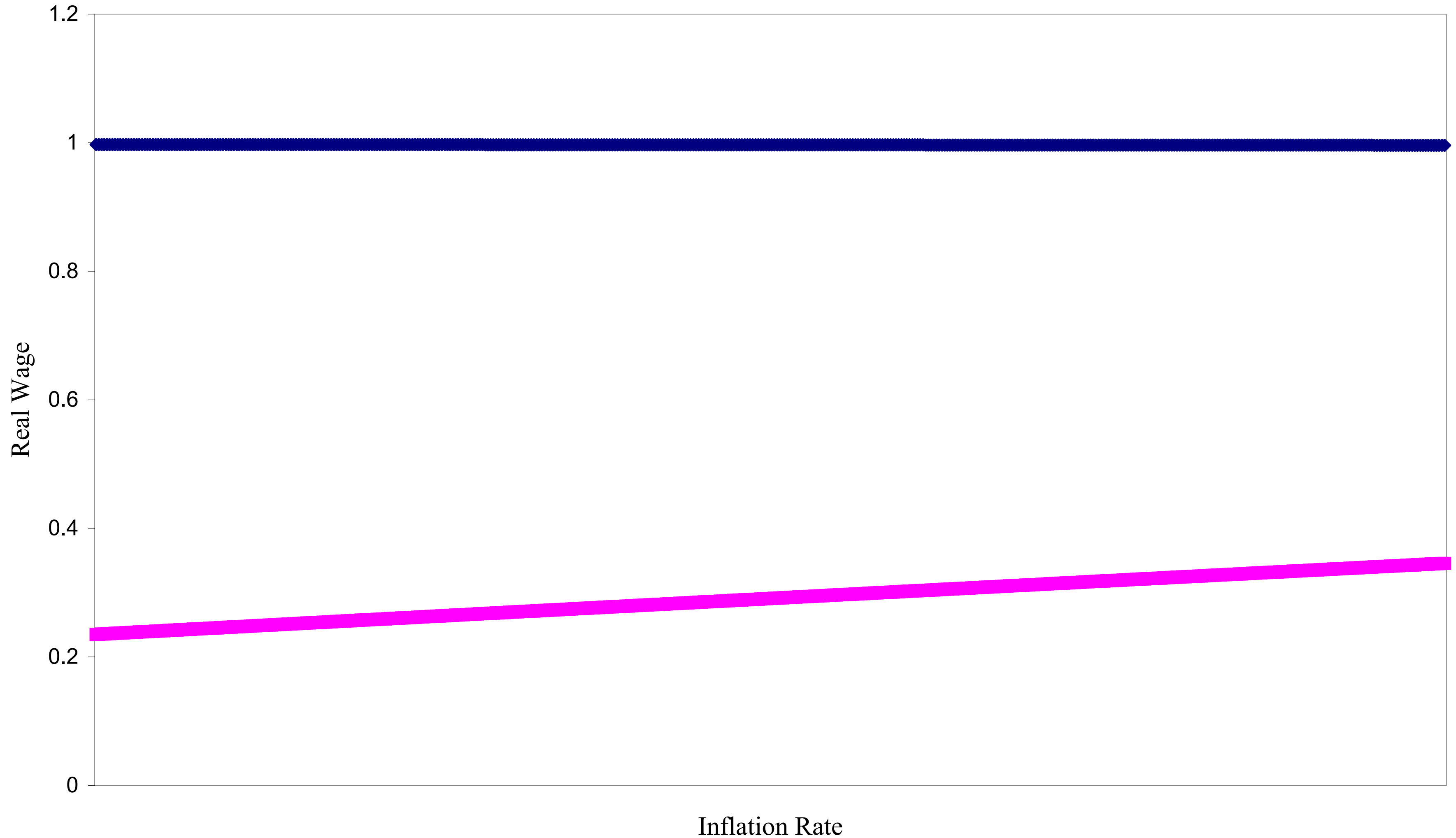

Figure 4
Effect of Inflation on the Average Real Wage Offer and Earnings

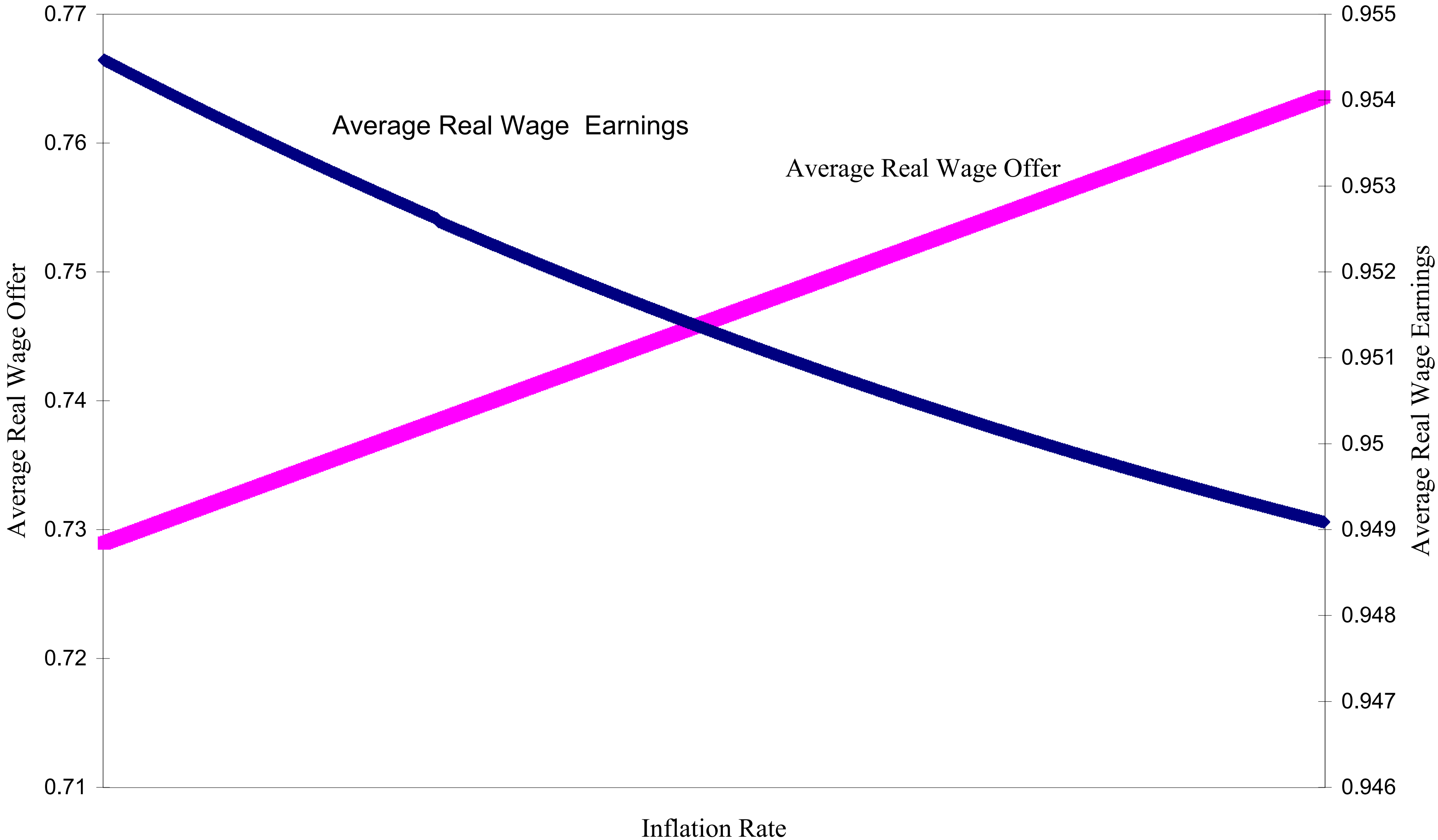

Figure 5
Effect of Inflation on Coefficient of Variation of Real Wage Offers and Earnings