

Licensing a New Product with Non-Linear Contracts[†]

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Abstract

This paper shows how a licensor that has developed a technology which permits the introduction of a new good can obtain the monopoly profit when licensing its innovation to firms competing à la Cournot. The licensor is able to obtain the monopoly profit by manipulating the equilibrium quantities of both the licensees and the nonlicensees using a licensing contract specifying a fixed fee and a royalty scheme despite the facts that the good initially produced is differentiated from the new good and that firms have initially different marginal cost of production. While the licensing contract does not result in the monopolization of the market, the initial good will no longer be produced.

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1 Introduction

Shapiro (1985) introduced the idea that a licensor of a cost-reducing technology could obtain the monopoly profit by designing a licensing contract specifying a judicially chosen royalty rate per unit of output. While Shapiro's intuition is correct, his analysis breaks down when the number of potential licensees is greater than one unless the innovation is drastic.¹ Erutku and Richelle (2000) extend Shapiro's intuition and show how a licensor of a cost-reducing technology can obtain the monopoly profit whatever the number of firms on the market and the quality of the innovation. To achieve this, the licensor must manipulate the equilibrium quantities of both the licensees and the nonlicensees such that the total production will be equal to the monopoly output and the reservation profit of the licensees will be equal to zero.

Here, this idea is applied in a slightly different context. Initially, two firms having different marginal cost and competing à la Cournot produce a homogeneous good. A licensor is able to develop a new technology which permits the introduction of a new good. The existing good and the new good are differentiated yet substitutable. Despite the cost asymmetries and the product differentiation, it is shown that the innovator is able to obtain the monopoly profit.

In order to perform our analysis, we follow a framework similar to the one developed by Kamien, Tauman, and Zang (1988) who restrict their analysis to licensing by means of a fixed fee. In their setting, they postulate that the introduction of a new product can be regarded as a cost reducing innovation. In effect, while the new product could have been produced, its marginal cost of production would have been sufficiently high to make its production unprofitable. In that sense, the innovation constitutes a reduction of the marginal cost of the new product.

We find that using a fixed fee and a royalty scheme, the licensor is always able to obtain the

¹A cost-reducing innovation is drastic if the monopoly price with the new technology is smaller than the competitive price with the old technology.

monopoly profit. However, this entails the disappearance of the existing good.

Thus, the model presented in this paper provides an explanation as to why all firms in an industry might switch to a new product when it is introduced following the development of a cost reducing technology (for example, computer manufacturers often switch to the new Intel pentium chips when they are released on the market).

Moreover, the model also provides an explanation as to why royalties are used in licensing contracts. In effect, here, the use of royalties comes from the licensor's desire to manipulate the equilibrium quantities of the licensees and nonlicensees in order to obtain the monopoly profit and not from the presence of *i*) asymmetric information (e.g., Hornsten (1998)), *ii*) uncertainty (e.g., Bousquet and al. (1998)), or *iii*) the innovator on the output market (e.g., Wang (1998)).

The paper is organized as follows. In Section 2, the model is presented. Section 3 describes how the licensor can design a contract to obtain the monopoly profit. Section 4 shows the influence of the degree of differentiation between the existing and the new good on the contractual terms. Finally, section 5 concludes.

2 Model

Consider an industry with two firms, firm 1 and firm 2. The demand side of the market is described by the preferences of a representative consumer over the set of available products. These preferences are represented by the following quadratic utility function:

$$U(Q_x, Q_y) = a(Q_x + Q_y) - (1/2)(Q_x^2 + Q_y^2 + 2\beta Q_x Q_y),$$

where Q_x stands for the total quantity purchased of the existing good, and Q_y stands for the total quantity purchased of the new good; $a > 0$ gives the absolute size of the market, and $\beta \in (0, 1]$ is an indicator of the degree of substitutability between the two products. If $\beta = 1$, then the products

are perfect substitutes; if $\beta = 0$, then the products are independent.

From the consumer's maximization problem, the linear inverse demand for products x and y are respectively:

$$p_x = a - Q_x - \beta Q_y,$$

$$p_y = a - Q_y - \beta Q_x.$$

Initially, the new good, y , is too costly to produce. A sufficient condition for this is that the constant marginal cost of the new good with the current technology, \bar{c} , is such that $\bar{c} > [a(2 - \beta) + \beta c_2]/2$, where c_2 is the constant marginal cost of the highest cost producer of the existing good.

Since it is too costly to produce the new good before the innovation, both firms produce the existing good with their respective technology. The constant marginal costs of production for firms 1 and 2 are c_1 and c_2 respectively, with $0 \leq c_1 < c_2$. If both firms produce strictly positive quantities, the inverse demand for the existing good is $p_x = a - Q_x$ with $Q_x = q_{1x} + q_{2x}$.

In addition to the two firms, there is an independent research lab, hereafter referred to as the licensor, who owns a patent for a technology that reduces the marginal cost of production of the new good from \bar{c} to $c \in [0, \frac{a(2-\beta)+\beta c_2}{2})$. Following this technological improvement, the production of the new good becomes profitable. Since the licensor is not a member of the industry, he only seeks to license his innovation to the firms so as to maximize his licensing revenues.

Interactions between the two firms and the licensor are described by the following three-stage game. In the first stage, the licensor offers a licensing contract that specifies an up-front fixed fee α , and a royalty scheme $\tau(q)$. The royalties paid by firm h will depend on firm h 's output, with $h = 1, 2$. Any contract includes a clause that prohibits a licensee from reselling his license and contracts are costlessly enforceable by courts.

At the second stage, the two firms observe the cost of producing the new good allowed by the

innovation, c , and the proposed contract (α, τ) . Then, they decide simultaneously to accept or to reject the contract. If a firm accepts the contract, it immediately pays the fixed fee α . The set of firms, $N = \{1, 2\}$, is therefore partitioned into two subsets: the set of licensees, denoted by L , and the set of nonlicensees, denoted by $N \setminus L$. We shall denote by l and k an element of L and $N \setminus L$ respectively, and we shall let $m = |L|$. The production of a licensee will be denoted by $q_{ly}(m, \tau)$, while the production of a nonlicensee will be denoted by $q_{kx}(m, \tau)$.²

At the third stage, each firm observes whether or not the other has accepted the proposed contract and they decide simultaneously how much to produce. After its production has been sold on the market, a licensee will pay the appropriate royalties. The profit of a licensee, π_{ly} , and the profit of a nonlicensee, π_{kx} , are respectively given by:

$$\begin{aligned}\pi_{ly} &= (a - q_{ly} - \beta q_{kx} - c)q_{ly} - \tau(q) - \alpha, \\ \pi_{kx} &= (a - q_{kx} - \beta q_{ly} - c_k)q_{kx}.\end{aligned}$$

The revenue of the licensor, it is given by:

$$\sum_{l \in L} [\alpha + \tau(q_{ly})].$$

We shall look only for a contract that gives the monopoly profit to the licensor.

3 Licensing contracts

In this section, we will show how a licensor, by designing a contract (α, τ) , can obtain the profit that a monopolist using its innovation would earn. This will be true for all values of the parameters a , β , c , c_1 , and c_2 . More specifically, we will consider contracts specifying sliding scale per-unit

²We assume that if a firm accepts the contract, it replaces the old technology by the new technology which cannot be used to produce the existing good.

royalties, i.e., contracts such that the royalty is of the form of $\tau(q) = (\rho - \mu q)q$ with $\rho \geq 0$ and $\mu \geq 0$.

To find such a contract, we must first analyze the last two stages of the game. At the third stage, after having observed the characteristics of the proposed contract, α , ρ , and μ , and the set of firms that have accepted it, L , the firms choose their output to maximize their profit.

If no firms accept the proposed contract (scenario 0), the first-order conditions (FOC) for profit maximization by firms 1 and 2 are respectively given by:

$$a - 2q_{1x} - q_{2x} - c_1 \leq 0 \text{ with equality if } q_{1x} > 0, \quad (1)$$

$$a - 2q_{2x} - q_{1x} - c_2 \leq 0 \text{ with equality if } q_{2x} > 0. \quad (2)$$

Since the FOC are independent of ρ and μ , and by abusing slightly the notation, the equilibrium Cournot quantities for firms 1 and 2 are respectively noted $q_{1x}^c(m, \rho, \mu) = q_{1x}^c(0, 0, 0)$ and $q_{2x}^c(m, \rho, \mu) = q_{2x}^c(0, 0, 0)$ and are supposed strictly positive. For $q_{1x}^c(0, 0, 0) > 0$, we need $(a - c_1) + (c_2 - c_1) > 0$. This condition is satisfied since, by our assumptions, $a > c_h \forall h = 1, 2$ and $c_2 > c_1$. For $q_{2x}^c(0, 0, 0) > 0$, we assume that $a - 2c_2 + c_1 > 0$.

If only one firm accepts the proposed contract (scenario 1), the FOC for profit maximization by a licensee and a nonlicensee are respectively given by:

$$(a - c) - (q_{ly} + \beta q_{kx}) - \rho - (1 - 2\mu)q_{ly} \leq 0 \text{ with equality if } q_{ly} > 0, \quad (3)$$

$$(a - c_k) - (q_{kx} + \beta q_{ly}) - q_{kx} \leq 0 \text{ with equality if } q_{kx} > 0. \quad (4)$$

The equilibrium Cournot quantities by a licensee and a nonlicensee are respectively noted $q_{ly}^c(1, \rho, \mu)$ and $q_{kx}^c(1, \rho, \mu)$ for any $l \in L$ and any $k \in N \setminus L$.

Finally, if both firms accept the proposed contract (scenario 2), the FOC are:

$$(a - c) - (q_{1y} + q_{2y}) - \rho - (1 - 2\mu)q_{1y} \leq 0 \text{ with equality if } q_{1y} > 0, \quad (5)$$

$$(a - c) - (q_{1y} + q_{2y}) - \rho - (1 - 2\mu)q_{2y} \leq 0 \text{ with equality if } q_{2y} > 0. \quad (6)$$

and the equilibrium Cournot quantities are noted $q_{ly}^c(2, \rho, \mu) \forall l = 1, 2$.

From the equilibrium quantities in each scenario, it is possible to compute the firms' Cournot equilibrium profit $\pi_{hg}^c(\cdot)$ (gross of the fixed fee α for the licensee(s)), with $h = 1, 2$ and $g = x, y$.

At the second stage of the game, firms simultaneously choose to accept or to reject the proposed contract. Suppose that one firm decides to accept the contract. The other firm will decide to accept the contract if the profit it will make as a licensee, $\pi_{ly}^c(2, \rho, \mu) - \alpha$, is greater than or equal to the profit it will make as a nonlicensee, $\pi_{kx}^c(1, \rho, \mu)$, i.e., if $\alpha \leq \pi_{ly}^c(2, \rho, \mu) - \pi_{kx}^c(1, \rho, \mu)$. Similarly, suppose that one firm has decided to reject the contract. The other firm will decide to accept the contract if the profit it will make as a licensee, $\pi_{ly}^c(1, \rho, \mu) - \alpha$, is greater than or equal to the profit it will make as a nonlicensee π_{kx}^c , i.e., if $\alpha \leq \pi_{ly}^c(1, \rho, \mu) - \pi_{kx}^c(0, 0, 0)$. More generally, in order to sell to m firms, the licensor will set the fixed fee such that $\alpha \leq \pi_{ly}^c(m, \rho, \mu) - \pi_{kx}^c(m-1, \rho, \mu)$.³ Since the licensor's revenue $\sum_{l \in L} [\alpha + \tau(q_{ly})]$ is increasing in α , we will have $\alpha = \pi_{ly}^c(m, \rho, \mu) - \pi_{kx}^c(m-1, \rho, \mu)$. We suppose that whenever a firm obtains the same profit by accepting or rejecting the proposed contract, it accepts the contract.

Thus, we can write the licensor's revenue as:

$$R = \sum_{l \in L} \{ \pi_{ly}^c(m, \rho, \mu) + [\rho - \mu q_{ly}^c(m, \rho, \mu)] q_{ly}^c(m, \rho, \mu) \} - \sum_{l \in L} \{ \pi_{kx}^c(m-1, \rho, \mu) \}. \quad (7)$$

The first term in (7) stands for the licensor's benefit, consists of the sum of the licensees' profits and the royalties paid by the licensees, and writes as:

$$B = \sum_{l \in L} \{ [a - \sum_{l \in L} q_{ly}^c(m, \rho, \mu) - \beta \sum_{k \in N \setminus L} q_{kx}^c(m, \rho, \mu) - c] q_{ly}^c(m, \rho, \mu) \}. \quad (8)$$

The second term in (7) is the sum of the licensees' reservation profits and is noted Π^R .

In the first stage of the game, the licensor, to obtain the largest revenue possible, will choose

³To obtain a positive licensing revenue, the licensor will license its technology to at least one firm. This means that $m-1$ will always be greater than or equal to 0.

the number of licensees, m , and the value of the sliding scale per-unit royalties, ρ and μ , such that the difference between B and Π^R is maximized.

Definition 1. Let Π^M stand for the profit that a monopolist with a marginal cost c achieves on the market, i.e., $\Pi^M = \max_Q [P(Q) - c]Q$ with $P(Q) = a - Q$. A licensing contract is said strongly optimal if i) it is proposed at a subgame perfect Nash equilibrium of the game, ii) it leads to a revenue of Π^M for the licensor, and iii) the proposition of this contract has a unique outcome.

In order to obtain the profit that a monopolist with a marginal cost c would earn, the licensor can design a contract such that its benefit, B , will be equal to the monopoly profit, and the licensees' reservation profit, Π^R , will be equal to zero.

First, for B to be equal to the monopoly profit, the licensor can choose a royalty scheme such that the sum of the licensees and nonlicensees' quantity, at the equilibrium, will correspond to the quantity produced by a monopoly with marginal cost c .

Second, for Π^R to be equal to zero, the licensor can choose a royalty scheme such that the profit of a nonlicensee will be equal to zero. In effect, a licensee's reservation profit is simply equal to a nonlicensee's profit with one less firm that has accepted the proposed contract. Since a nonlicensee's profit in a Cournot game can be expressed in terms of quantity, the royalty scheme must be such that no nonlicensees produce strictly positive quantity at the equilibrium.

Third, since we are looking for a subgame perfect Nash equilibrium (SPNE), no firm must have an incentive to deviate unilaterally.

The requirements needed for the licensor to earn the monopoly profit can be summarized by the following six conditions:

1. whenever m firms have decided to accept the contract, the sum of the licensees' quantity must be equal to the monopoly quantity: $\sum_{l \in L} q_{ly}^c(m, \rho, \mu) = Q^M$,

2. whenever m firms have decided to accept the contract, no nonlicensee produces a strictly positive quantity: $q_{kx}^c(m, \rho, \mu) = 0 \forall k \in N \setminus L$,
3. whenever $m - 1$ firms have decided to accept the contract, no nonlicensee produces a strictly positive quantity: $q_{kx}^c(m - 1, \rho, \mu) = 0 \forall k \in N \setminus L$,
4. whenever $m - 1$ firms have decided to accept the contract, it must be profitable for another firm to accept the contract: $\alpha \leq \pi_{ly}^c(m, \rho, \mu) - \pi_{kx}^c(m - 1, \rho, \mu)$,
5. whenever m firms have decided to accept the contract, it must not be profitable for another firm to accept the contract: $\alpha > \pi_{ly}^c(m + 1, \rho, \mu) - \pi_{kx}^c(m, \rho, \mu)$,
6. whenever m firms have decided to accept the contract, the marginal cost of the licensees must be greater than or equal to zero: $c + \rho - 2\mu q_{ly}^c(m, \rho, \mu) \geq 0$.

With this in hand, we can determine how m , ρ , μ , and α will be chosen by the licensor in order to design a strongly optimal contract.

Initially, it is best to look at the choice of the number of licensees. To do so, consider the third condition that must be fulfilled for a contract to be strongly optimal. If only one firm accepts the proposed contract, then $m - 1 = 0$. Since the equilibrium quantities in scenario 0 are independent of both ρ and μ , and are supposed positive, then, if only one firm, say h , becomes a licensee, its reservation profit will be $[q_{hx}^c(0, 0, 0)]^2 > 0$ and the licensor's revenue will be less than the monopoly profit. Thus, to design a strongly optimal contract, the licensor must sell to both firms, i.e., $m = 2$.

The rest of this section will show how ρ , μ , and α will be chosen such that *i*) the combined output of the licensees will equal the monopoly output, *ii*) the reservation profit of the licensees will equal zero, and *iii*) the licensor will be able to extract the monopoly profit from the licensees.

To satisfy condition 1, we must have $\sum_{l \in L} q_{ly}^c(2, \rho, \mu) = Q^M = (a - c)/2$. Since firms are

symmetric, $q_{ly}^c(2, \rho, \mu) = Q^M/2 \forall l \in L$. Therefore, we can rewrite (5) and (6) as:

$$(a - c) - \left(\frac{a - c}{2}\right) - \rho - (1 - 2\mu) \left(\frac{a - c}{4}\right) = 0. \quad (9)$$

Solving for ρ , we obtain:

$$\rho = \frac{(a - c)(1 + 2\mu)}{4}. \quad (10)$$

Since the licensor sells to both firms in the industry, condition 2 is automatically satisfied.

To satisfy condition 3, we must have $q_{kx}^c(1, \rho, \mu) = 0$. Using (3) and (4), $q_{kx}^c(1, \rho, \mu) = 0$ if and only if ρ is such that:

$$\rho \leq \frac{\beta(a - c) - 2(1 - \mu)(a - c_k)}{\beta}. \quad (11)$$

Now, setting (10) equal to (11), we find that conditions 1 and 3 are satisfied if ρ is equal to (10) and μ is such that:

$$\hat{\mu} = \frac{8K_k - 3\beta}{8K_k - 2\beta} \quad (12)$$

where $K_k = (a - c_k)/(a - c)$.

Moreover, replacing (11) in either (5) or (6) and using the fact that firms are symmetric when they both accept the proposed contract, we have:

$$q_{ly}^c(2, \rho, \mu) = \frac{2(1 - \mu)(a - c_k)}{\beta(3 - 2\mu)} > 0 \forall l = 1, 2 \text{ if } \mu < 1.$$

It is easy to verify that $\hat{\mu} < 1$ and that $\hat{\mu} \geq 0$ if and only if $K_k \geq 3\beta/8$. Hence, we have:

$$\hat{\mu} = \begin{cases} \frac{8K_k - 3\beta}{8K_k - 2\beta} & \text{if } K_k \geq 3\beta/8 \\ 0 & \text{if } K_k < 3\beta/8 \end{cases} \quad (13)$$

Finally, replacing μ by (13)⁴ in (10), we have:

$$\hat{\rho} = \begin{cases} \frac{(a-c)(3K_k - \beta)}{4K_k - \beta} & \text{if } K_k \geq 3\beta/8 \\ \frac{a-c}{4} & \text{if } K_k < 3\beta/8 \end{cases} \quad (14)$$

For $K_k \geq 3\beta/8$, both (13) and (14) depend on the level of c_k . Hence, we need to determine whether this level of c_k is c_1 or c_2 . The reason why $\hat{\rho}$ and $\hat{\mu}$ are function of c_k is because the licensor wants to set $q_{kx}^c(1, \rho, \mu) = 0$, i.e., wants to set the reservation profit of the licensees equal to zero. Because $c_1 < c_2$, whenever $q_{1x}^c(1, \rho, \mu) = 0$, $q_{2x}^c(1, \rho, \mu)$ will also be equal to zero. However, the reverse is not true. Therefore, $c_k = c_1$ which implies that:

$$\mu^* = \begin{cases} \frac{8K_1 - 3\beta}{8K_1 - 2\beta} & \text{if } K_1 \geq 3\beta/8 \\ 0 & \text{if } K_1 < 3\beta/8 \end{cases} \quad (15)$$

and

$$\rho^* = \begin{cases} \frac{(a-c)(3K_1 - \beta)}{4K_1 - \beta} & \text{if } K_1 \geq 3\beta/8 \\ \frac{a-c}{4} & \text{if } K_1 < 3\beta/8 \end{cases} \quad (16)$$

with $K_1 = (a - c_1)/(a - c)$.

To satisfy condition 4, the fixed fee α must be such that whenever one firm has decided to accept the contract, it must be profitable for the other firm to accept it. This means that the fixed fee will be set such that $\alpha = \pi_{ly}^c(2, \rho, \mu) - \pi_{kx}^c(1, \rho, \mu)$ where:

$$\pi_{kx}^c(1, \rho, \mu) = [a - q_{kx}^c(1, \rho, \mu) - \beta q_{ly}^c(1, \rho, \mu) - c_k] q_{kx}^c(1, \rho, \mu).$$

In order to fulfill condition 3, ρ^* and μ^* were chosen such that $q_{kx}^c(1, \rho, \mu) = 0$. Therefore,

$$\alpha^* = \pi_{ly}^c(2, \rho^*, \mu^*) \quad (17)$$

which is equal to half the monopoly profit by condition 1.

⁴When both firms accept the proposed contract, equation (13) ensures that the profit function of a licensee is concave in q_{ly} since $\partial^2 \pi_{ly} / \partial q_{ly}^2 = -2(1 - \mu)$.

With only two firms in the industry, condition 5 is automatically satisfied.

Finally, to show that the marginal cost of the licensees will always be strictly greater than zero, two cases must be considered.

Case 1. Suppose that $K_1 > 3\beta/8$. This means that $\rho^* = (a - c)(3K_1 - \beta)/(4K_1 - \beta)$, $\mu^* = (8K_1 - 3\beta)/(8K_1 - 2\beta)$, and $q_{ly}^c(2, \rho, \mu) = (a - c)/4$. Replacing these in $c + \rho - 2\mu q_{ly}^c(2, \rho, \mu)$, we find that the marginal cost of the licensees is always greater than zero if $K_1 > \beta/4$. Since, in this case $K_1 > 3\beta/8$, the condition is satisfied.

Case 2. Suppose that $K_1 \leq 3\beta/8$. This means that $\rho^* = (a - c)/4$, $\mu^* = 0$, and $q_{ly}^c(2, \rho, \mu) = (a - c)/4$. Replacing these in $c + \rho$, we find that the marginal cost of the licensees is equal to $a + 3c$ which is always greater than zero.

It is possible to illustrate graphically the intuition behind the results of the above analysis. Figure 1 illustrates the case where both firms are licensees. The reaction function of firms 1 and 2 in space (q_{1y}, q_{2y}) are respectively given by:

$$R_{1y}(2, \rho, \mu) = (a - c - \rho) - 2(1 - \mu)q_{1y}, \quad (18)$$

$$R_{2y}(2, \rho, \mu) = \frac{a - c - \rho}{2(1 - \mu)} - \frac{q_{1y}}{2(1 - \mu)}. \quad (19)$$

When $\rho = \mu = 0$, the reaction functions of firms 1 and 2, respectively noted $R_{1y}(2, 0, 0)$ and $R_{2y}(2, 0, 0)$, cross at a point corresponding to the Cournot equilibrium when both firms have marginal cost c . To satisfy condition 1, the licensor, for a given value of μ , sets ρ to a positive value equal to the right-hand side of (10). Given the equilibrium quantities, this shifts both reaction functions to the left where they cross at a point where the combined production of both firms coincide to the output of a monopoly with marginal cost c . This does not ensure the monopoly profit to the licensor. In effect, he also needs to set the firms' reservation profit to zero. Figure

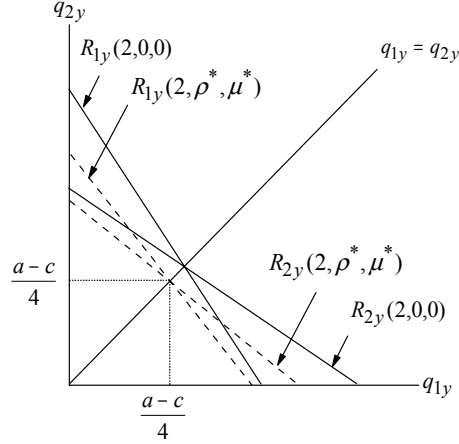


Figure 1: Condition 1

2 illustrates the case where only one firm is a licensee. Suppose that firm 1 has accepted the proposed contract while firm 2 has not. The reaction function of firms 1 and 2 in space (q_{1y}, q_{2x}) are respectively given by:

$$R_{1y}(2, \rho, \mu) = \frac{a - c - \rho}{\beta} - \frac{2(1 - \mu)}{\beta} q_{1y}, \quad (20)$$

$$R_{2x}(2, \rho, \mu) = \frac{a - c}{2} - \frac{\beta}{2} q_{1y}. \quad (21)$$

For a given value of ρ , the reaction functions of the licensee and nonlicensee cross at a point where the output of both firms are strictly positive. To satisfy condition 3, the licensor sets μ such that, given the equilibrium quantities, the reaction function of the licensee is tilted until it crosses the reaction function of the nonlicensee where the latter produces nothing.

By simultaneously choosing ρ and μ the licensor is able to satisfy conditions 1 and 3.

To conclude this section, we must turn our attention to particular cases where there does not necessarily exist strongly optimal contracts.

Case 1. Suppose that $\mu^* = 1/2$. From (15), this would be the case if $K_1 = \beta/2$. It would mean that $\rho^* = (a - c)/2$, and that (18) is equal to (19). The infinity of Cournot equilibria would imply that the proposed contract would not be a strongly optimal contract since there would not

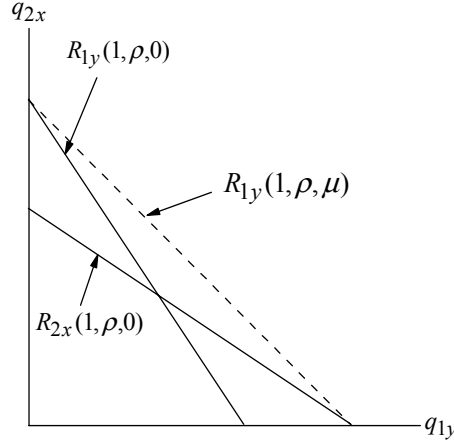


Figure 2: Condition 3

be a unique equilibrium outcome.

Case 2. Suppose that $\mu^* = 1/2$, $K_1 = \beta/2$, and $\rho^* = (a-c)/2$ as in case 1. Since $K_1 = \beta/2$, the reaction functions of the licensee and the nonlicensee cross each other once at $q_{kx} = 0$ and $q_{ly} = Q^M$. However, this cannot be a strongly optimal contract since $q_{ly}(m-1, \rho^*, \mu^*) > 0$ meaning that the reservation profit of the licensee is greater than zero.

Case 3. There exists a Cournot equilibrium at which some licensee does not produce. To show this, assume that, say, $q_{2y}^c(2, \rho^*, \mu^*) = 0$. We can then obtain from equation (5):

$$q_{1j} = \frac{a - c - \rho}{2(1 - \mu)}$$

which, once included in (6), gives:

$$\frac{(1 - 2\mu)(a - c - \rho)}{2(1 - \mu)} - 2(1 - \mu)q_{2y}. \quad (22)$$

We want to find the sign of (22). First, we know that $\mu < 1$ and that $q_{2y} \geq 0$. This means that the second term of (22) is greater than or equal to zero. Second, replacing ρ by (10), the first term of (22) can be written as $(1 - 2\mu)(a - c)(3 - 2\mu)/8(1 - \mu)$ which is lower than zero whenever $K_1 > \beta/2$ because μ becomes greater than $1/2$. Therefore, since the FOC of the last firm is strictly negative,

we have a Cournot equilibrium where one of the firms does not produce. To eliminate this kind of equilibrium, the licensor can impose a strictly positive penalty f if any licensee does not produce.

4 Comparative statics

This section investigates the role of the degree of product differentiation in the choice of the strongly optimal contract, i.e., ρ^* and μ^* . As it can be seen from (5) and (6), the FOC of the licensees are independent of β when both firms accept the contract. This is because both firms produce the new good. Moreover, to satisfy condition 1, ρ^* and μ^* are chosen such that the reaction functions of the licensees cross at half the monopoly quantity. This was depicted in Figure 1.

However, when only one firm accepts the proposed contract, the reaction functions of the licensee and the nonlicensee, respectively given by (20) and (21), are function of β . This implies that the degree of differentiation between the existing and the new good will influence the choice of ρ^* and μ^* . Moreover, to satisfy condition 3, ρ^* and μ^* are chosen such that the nonlicensee does not produce a strictly positive quantity. This was depicted in Figure 2.

To determine how ρ^* and μ^* will be affected by a change in β , we simply have to partially differentiate (16) and (15) with respect to β :

$$\begin{aligned}\frac{\partial \rho^*}{\partial \beta} &= -\frac{K_1(a-c)}{(4K_1-\beta)^2} < 0, \\ \frac{\partial \mu^*}{\partial \beta} &= -\frac{2K_1}{(4K_1-\beta)^2} < 0.\end{aligned}$$

The intuition for this result is the following. Suppose that we start from an initial value of β , say $\underline{\beta}$. When only one firm accepts the proposed contract, say firm 1, the two reaction curves, $R_{1y}(1, \rho^*, \mu^*)$ and $R_{2x}(1, \rho^*, \mu^*)$, cross at a point where $q_{2x}^c(1, \rho^*, \mu^*) = 0$ fulfilling condition 3. Now, if the degree of product differentiation were to increase, to say $\bar{\beta}$, it would have the following effect: *i*) the intercept and the slope (in absolute value) of the reaction function of the licensee

would respectively be unchanged and increase, and *ii*) the intercept and the slope (in absolute value) of the reaction function of the nonlicensee would both decrease. If the parameters of the royalty scheme were not adjusted, condition 3 would be violated, i.e., $q_{2x}^c(1, \rho^*, \mu^*) > 0$. In effect, as competition intensifies with the increase in β (because the goods become more substitutable), the licensee would suffer from a relative cost disadvantage if the parameters of the royalty scheme were not adjusted. Therefore, following a variation in β , the licensor would adjust ρ^* and μ^* in order to restore the desired outcome.

While the adjustments in ρ^* and μ^* would not affect the reaction function of the nonlicensee, it would decrease the intercept and the slope (in absolute value) of the reaction function of the licensee. The change in the intercept of the reaction function of the licensee would exactly match the change in the intercept of the reaction function of the nonlicensee following the increase in β so that both reaction functions would cross, once again, at $q_{2x}^c(1, \rho^*, \mu^*) = 0$.

5 Conclusion

In this paper, we address the question of how the inventor of a new technology which permits the introduction of a new good can design a licensing contract to obtain the monopoly profit despite the presence of an existing differentiated and substitutable good. Depending on the marginal cost of producing the new good with the cost reducing innovation, the contract specifies either a linear royalty per unit of output or a sliding-scale per unit royalty which are used by the innovator to manipulate the equilibrium quantities of both the licensees and the nonlicensees. While the licensing contract does not result in the monopolization of the market, the initial good will no longer be produced since all firms will become licensees. Thus, this analysis suggests that a licensing contract can induce all the firms in the industry to abandon the existing good in favor of the new good.

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